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Arrested Development: Theory and Evidence of Supply-Side Speculation in the Housing Market

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ABSTRACT

This paper studies the role of disagreement in amplifying housing cycles. Speculation is easier in the land market than in the housing market due to frictions that make renting less efficient than owner-occupancy. As a result, undeveloped land facilitates construction and intensifies the speculation that causes booms and busts in house prices. This observation challenges the standard intuition that in cities where construction is easier, house price booms are smaller. It can also explain why the largest house price booms in the United States between 2000 and 2006 occurred in areas with elastic housing supply.

ASSET PRICES GO THROUGH PERIODS of sustained price increases, followed by busts. To explain these episodes, economists have developed theories based on disagreement, speculation, and strategic trading. This literature focuses on the behavior of asset prices in stock markets. But do these ideas explain housing markets as well? Like any other financial asset, housing is a traded, durable claim on uncertain dividends. Moreover, an enduring feature of housing markets is booms and busts in prices that coincide with widespread disagreement about fundamentals (Shiller (2005)), and there is a long history of investors using real estate to speculate about the economy (Kindleberger (1978), Glaeser (2013)).

In this paper, we incorporate disagreement into a housing model to examine whether the finding that disagreement raises stock prices generalizes to

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the housing market.[1] Housing differs from the typical asset studied in finance in two fundamental ways. First, the underlying real assets in many financial models are in fixed supply.[2] In contrast, elastic supply is central to housing markets, as firms respond to high prices with new construction (Gyourko (2009)). Second, while the typical financial asset pays cash dividends, the dividend paid by housing is the flow utility enjoyed by end users. This flow utility has different values for different people and is not perfectly transferable, leading many people to prefer owning to renting (Henderson and Ioannides (1983)).

We study a two-period model of a housing market with two classes of agents. Potential residents receive heterogeneous utility from consuming housing that accrues only when they own their houses. Developers supply housing in a competitive market, buying land at market prices, and converting it into housing for a constant resource cost. As in Saiz (2010), the amount of developable land is fixed due to geographic and regulatory constraints. At $t = 0$, there is an initial number of potential residents N_0, and at $t = 1$, the number of potential residents grows to $N_1 = e^{\mu x} N_0$, where x is a positive shock. The larger is N_1, the greater are the price of land and housing at $t = 1$. At $t = 0$, all agents observe the shock x, but they "agree to disagree" about μ. In this context, we define a "house price boom" as the reaction of the house price at $t = 0$ to the shock x.

Our results characterize how the size of the house price boom varies with N_0. When N_0 is very small, there is no house price boom because the price of land remains equal to zero. When N_0 is very large, initial housing demand can be so strong that, at $t = 0$, all space is held by homeowners. In this case, the house price at $t = 0$ reflects both heterogeneous expectations about the $t = 1$ price and homeowners' flow utility for housing. The latter mitigates the impact of the former. Finally, when N_0 is intermediate, some land at $t = 0$ is left unoccupied by homeowners and is held by developers. These *supply-side speculators* hold land only when they have optimistic beliefs about μ, and it is these optimistic beliefs that drive prices. Consequently, prices at $t = 0$ are most sensitive to disagreement and optimism for cities with an intermediate level of development. For the same reason, the house price at $t = 0$ is most sensitive to x, that is, booms are larger, in these cities.

Stated in terms of available land in the city, the house price boom is largest for intermediate values of initial land supply. This nonmonotonicity between the house price boom and supply contrasts with prior work on disagreement—which does not consider the unique aspects of housing—and prior work on

[1] Beginning with Miller (1977), a large literature uses models of disagreement to explain asset pricing patterns in the stock market. Hong and Stein (2007) survey this literature, which includes Harrison and Kreps (1978), Morris (1996), Diether, Malloy, and Scherbina (2002), Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006), and Simsek (2013). Other papers apply speculative finance models to housing. See, for example, Piazzesi and Schneider (2009), Favara and Song (2014), Giglio, Maggiori, and Stroebel (2014), and Burnside, Eichenbaum, and Rebelo (2015). Unlike those papers, our work focuses on housing supply.

[2] Although firms can (and do) issue new equity in response to high demand for stock and high prices (Baker and Wurgler (2002)), many asset pricing models assume that the underlying real assets of a firm are fixed. See, for instance, Lucas (1978) or Scheinkman and Xiong (2003).

housing cycles—which does not consider disagreement. Taken separately, each approach predicts a monotonically declining relationship between a house price boom and initial land supply (Hong, Scheinkman, and Xiong (2006), Glaeser, Gyourko, and Saiz (2008), Paciorek (2013)).[3] By joining these approaches, we provide a new insight about housing cycles that neither approach offers alone.

We demonstrate the robustness of this result in several extensions that relax the model's assumptions in different ways. First, we consider the case in which developers can issue equity and investors can short-sell that equity. Second, we consider an extension in which landlords can speculate in the housing market and rent housing to pessimistic residents. In a final extension, we generalize the model to the case in which supply elasticity declines continuously with the level of initial demand. We also formally show how disagreement reduces welfare (in the sense of Brunnermeier, Simsek, and Xiong (2014)) by reallocating space from high-flow-utility pessimists to low-flow-utility optimists and developers.

The model's core insight helps explain the variation in 2000 to 2006 house price booms across U.S. cities. As shown by Davidoff (2013), a static supply-demand framework with a common national demand shock cannot account for cities in the "sand states" of Arizona, California, Florida, and Nevada that experienced strong price and quantity growth.[4] One possibility is that the demand shocks in these cities were especially large due to local credit conditions, differences in productivity and amenities, or heightened speculative activity by homebuyers (Barlevy and Fisher (2011), Davidoff (2013), Gao, Sockin, and Xiong (2016)). Our model offers an alternative explanation: house prices were more sensitive to a demand shock in these cities because the cities were at an intermediate level of development, and market participants disagreed about future prices.

We offer several pieces of empirical evidence in support of this explanation. Land price growth from 2000 to 2006 was high and closely correlated with house price growth across cities, whereas construction cost growth was not. Matching this fact distinguishes our model from Gao, Sockin, and Xiong (2015), who offer a theory of nonmonotonic house price booms that is agnostic on the differential roles of land prices and construction costs. We also document land market speculation from U.S. public home builders. These firms amassed land far in excess of their immediate construction needs during the boom, while investors short-sold home builder stocks more than nearly every other industry. The model predicts these outcomes for developers in intermediate cities only in the presence of disagreement.

Although we do not systematically examine land constraints in sand state cities, the case of Las Vegas offers a stark illustration of our model. Las Vegas is surrounded by land owned by the federal government, and Congress passed

[3] Simsek (2013) predicts that the introduction of new nonredundant financial assets increases the "speculative variance" of portfolio values, but this result does not apply to our findings because the price of undeveloped land is perfectly correlated with the price of housing in our equilibrium.

[4] Understanding the factors driving these outlier cities is crucial for evaluating research instrumenting for price growth with supply elasticity. Davidoff (2016) discusses problems with this instrument.

a law in 1998 prohibiting the sale of land outside a development ring depicted in Figure 1. In 2007, some people believed that the remaining land would be fully developed by 2017. During the boom, land prices rose from \$150,000 to \$700,000 per acre before losing the gains in the bust. In a striking example of supply-side speculation, a single land development fund, Focus Property Group, won the majority of the large land parcels auctioned between 2002 and 2005 by the federal government in Las Vegas, obtaining 5% of the undeveloped land remaining within the barrier.[5]

The paper proceeds as follows. Section I presents the core model. Sections II and III characterize equilibrium with agreement and disagreement, respectively. Section IV presents extensions of the core model. Section V analyzes the 2000 to 2006 U.S. housing boom and subsequent bust. Section VI concludes. An appendix contains all proofs of material in Sections I, II, and III; the Internet Appendix[6] provides proofs of material in Section IV.

I. A Housing Market with Disagreement

A. Housing Supply and Developer Demand

The city we study has a fixed amount of space S. At the beginning of period 0, all of this space exists as undeveloped land that can be used for housing. Housing and land trade in spot markets each period but cannot be sold short.[7] The price of land and housing at t are p_t^l and p_t^h, respectively.

Developers are private firms endowed with the entire supply of land at the beginning of period 0. Developers can borrow or lend freely in global capital markets at an interest rate normalized to zero. In each period, a developer makes three decisions: how much land to buy or sell, how much housing to build, and how much housing to sell. Building a unit of housing requires one unit of land and a resource cost k_t. To simplify equations and keep the user cost of housing constant over time, we set $k_0 = 2k$ and $k_1 = k$ for some $k > 0$.[8] At $t = 1$, the owners of each developer receive the proceeds from liquidation.

Each developer maximizes its subjective expectation of its liquidation value. Denote the holdings of housing, land, and bonds at the beginning of t by H_t, L_t, and B_t, respectively. Denote the control variables of home sales, land purchases, and home construction at each t by H_t^{sell}, L_t^{buy}, and H_t^{build}, respectively. At $t = 1$, the liquidation value π of a developer is the outcome of the constrained

[5] The sources for this paragraph include price data from Applied Analysis, auction records from the Bureau of Land Management, news reports in the *Las Vegas Review Journal* and the *New York Times*, and land-use information from the Southern Nevada Regional Planning Commission.

[6] The Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

[7] Short-sale constraints in the housing market result from a lack of asset interchangeability. Although housing is homogeneous in the model, empirical housing markets involve large variation in characteristics across houses. This variation in characteristics makes it essentially impossible to cover a short.

[8] We abstract from the possibility of overbuilding by allowing developers to build negative amounts of housing by turning a house into land, thereby recouping k_t.

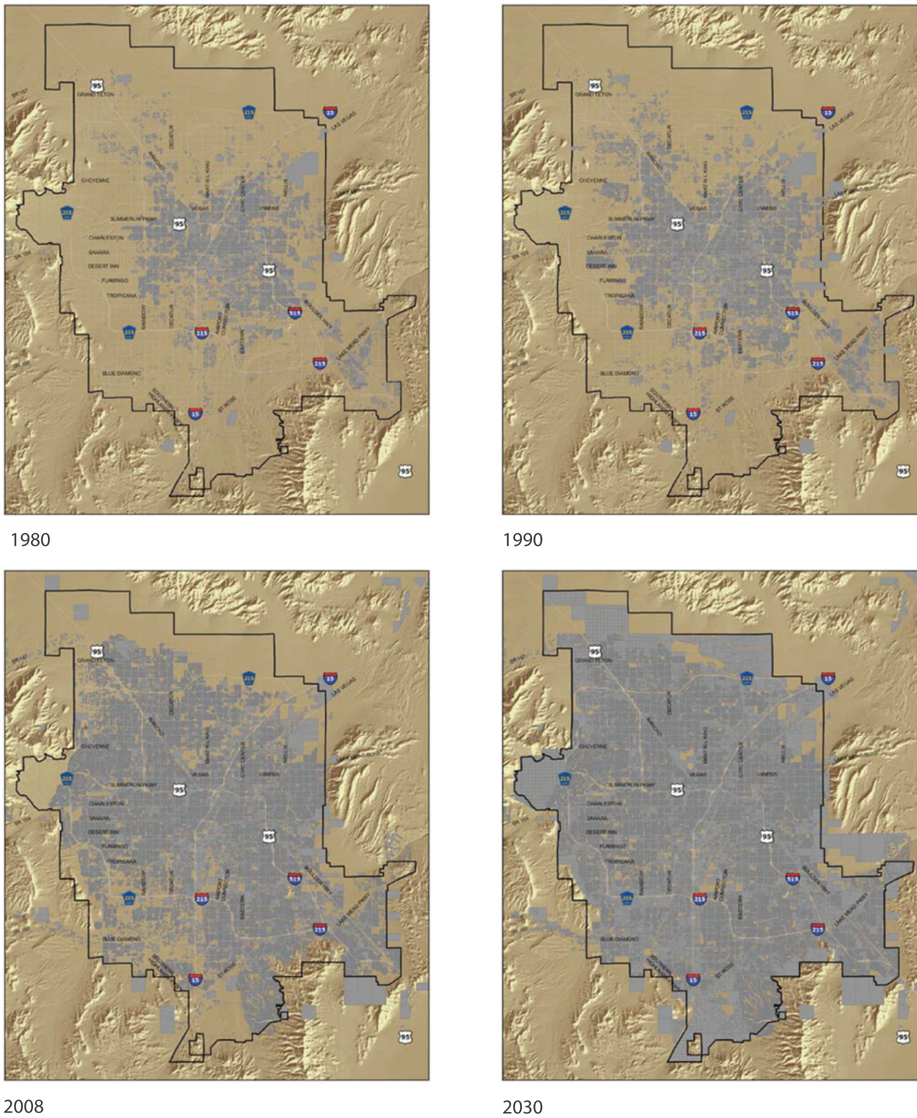

Figure 1. Long-run development constraints in Las Vegas. This figure comes from page 51 of the Regional Transportation Commission of Southern Nevada's *Regional Transportation Plan 2013–2035*. (Available at http://www.rtcsnv.com/wp-content/uploads/2012/10/Final_RTP-2013-35-Redetermination-0214131.pdf.) The first three pictures display the Las Vegas metropolitan area in 1980, 1990, and 2008. The final picture represents the Regional Transportation Commission's forecast for 2030. The boundary is the development barrier stipulated by the Southern Nevada Public Land Management Act. The shaded gray region denotes developed land. (Color figure can be viewed at wileyonlinelibrary.com)

optimization problem

$$\pi\left(p_1^h, p_1^l, H_1, L_1, B_1\right) = \max_{H_1^{\text{sell}}, L_1^{\text{buy}}, H_1^{\text{build}}} p_1^h H_1^{\text{sell}} - p_1^l L_1^{\text{buy}} - k H_1^{\text{build}} + B_1 \tag{1}$$

subject to

$$H_1^{\text{sell}} \leq H_1 + H_1^{\text{build}}, \tag{2}$$

$$H_1^{\text{build}} \leq L_1 + L_1^{\text{buy}}. \tag{3}$$

The actions $(H_1^{\text{sell}})^*$, $(L_1^{\text{buy}})^*$, and $(H_1^{\text{build}})^*$ chosen by the developer maximize this problem. At $t = 0$, the developer maximizes its subjective expectation of this liquidation value:

$$\left(H_0^{\text{sell}}\right)^*, \left(L_0^{\text{buy}}\right)^*, \left(H_0^{\text{build}}\right)^* \in \underset{H_0^{\text{sell}}, L_0^{\text{buy}}, H_0^{\text{build}}}{\arg\max}\ \mathrm{E}\pi\left(p_1^h, p_1^l, H_1, L_1, B_1\right) \tag{4}$$

subject to

$$H_0^{\text{sell}} \leq H_0^{\text{build}}, \tag{5}$$

$$H_0^{\text{build}} \leq L_0 + L_0^{\text{buy}}, \tag{6}$$

$$H_1 = H_0^{\text{build}} - H_0^{\text{sell}}, \tag{7}$$

$$L_1 = L_0 + L_0^{\text{buy}} - H_0^{\text{build}}, \tag{8}$$

$$B_1 = p_0^h H_0^{\text{sell}} - p_0^l L_0^{\text{buy}} - 2k H_0^{\text{build}}. \tag{9}$$

At $t = 0$, developers may differ only in their land endowments L_0 and their beliefs about p_1^h and p_1^l (as specified below). The sum of the land endowments across developers equals S. Developers take prices as given, which is consistent with evidence in Section V on perfect competition in the home building industry.

B. Individual Housing Demand

Potential residents derive utility from consumption and from owning and occupying housing. There are two disjoint groups of potential residents: one arriving at $t = 0$ and one arriving at $t = 1$. Upon arrival, each potential resident decides whether to buy a house. Utility comes from consumption c at $t = 1$ and any housing services v received in the period of arrival. Utility is linear and separable in housing and consumption: $u = c + v$ if the potential resident owns a house in the period of her arrival, and $u = c$ otherwise.

A potential resident who buys at $t = 0$ decides whether to sell her house at $t = 1$. As with developers, potential residents at $t = 0$ may borrow in global capital markets at an interest rate of zero. Denote the control variables capturing whether to buy or sell a house by H_t^{buy} and H_t^{sell}, each of which equals 0 or 1. At $t = 1$, an arriving potential resident chooses

$$\left(H_1^{\text{buy}}\right)^* \in \arg\max_{H_1^{\text{buy}}} H_1^{\text{buy}} \left(v - p_1^h\right) \tag{10}$$

subject to

$$H_1^{\text{buy}} \in \{0, 1\}, \tag{11}$$

and the utility of potential residents who bought at $t = 0$ equals

$$u\left(p_1^h, B_1, v\right) = \max_{H_1^{\text{sell}}} H_1^{\text{sell}} p_1^h + B_1 + v \tag{12}$$

subject to

$$H_1^{\text{sell}} \in \{0, 1\}, \tag{13}$$

where the choice $(H_1^{\text{sell}})^*$ maximizes this problem. At $t = 0$, arriving potential residents maximize the subjective expectation of their utility:

$$\left(H_0^{\text{buy}}\right)^* \in \arg\max_{H_0^{\text{buy}}} H_0^{\text{buy}}\, \mathrm{E}u\left(p_1^h, B_1, v\right) \tag{14}$$

subject to

$$H_0^{\text{buy}} \in \{0, 1\} \tag{15}$$

$$B_1 = -p_0^h H_0^{\text{buy}}. \tag{16}$$

At $t = 0$, potential residents may differ only in their housing utility v and in their beliefs about p_1^h (as specified below). At $t = 1$, arriving potential residents may differ only in v.

Denote by $D(v)$ the complementary cumulative distribution function of v among arriving potential residents. This time-invariant function encodes heterogeneity in housing flow utility. We make the following assumption about the functional form of $D(v)$:

ASSUMPTION 1: *There exists $\epsilon > 0$ such that*

$$D(v) = \begin{cases} 1 & \text{if } v < k \\ (k/v)^{\epsilon} & \text{if } v \geq k. \end{cases} \tag{17}$$

By Assumption 1, no potential residents have housing utility v less than k. This restriction implies that all potential residents are willing to purchase housing at its cost. As a result, no residents buy housing only because of expected capital gains. In the model, such pure speculators would be classified as developers.[9] Assumption 1 also invokes a constant elasticity of demand for housing, which allows us to derive simple analytic results.

Our utility specification makes two implicit assumptions about resident behavior. First, because utility is separable and linear in c, potential residents are risk-neutral. As a result, the purchase decisions of potential residents at $t = 0$ are not affected by the type of hedging motives studied by Piazzesi, Schneider, and Tuzel (2007). Second, because potential residents receive utility from only one house, their housing utility displays diminishing marginal returns. This property leads homeownership to be dispersed among residents in equilibrium.[10]

C. Aggregate Demand and Beliefs

Aggregate resident demand for housing depends on the number of potential residents and the joint distribution of housing utility and beliefs. The number of arriving potential residents at t equals $N_t S$, where $N_t > 0$. The growth in N_t between $t = 0$ and $t = 1$ is given by

$$\log(N_1/N_0) = \mu^{\text{true}} x, \tag{18}$$

where $x \geq 0$ is a shock and μ^{true} is some constant. At $t = 0$, all agents observe N_0 and x. They do *not* observe μ^{true}, the data needed to map the information shock x to the demand growth rate. Agents learn the value of μ^{true} at $t = 1$. The resolution of uncertainty at $t = 1$ is common knowledge at $t = 0$.

At $t = 0$, agents may disagree about the value of μ^{true}. Agent beliefs at $t = 0$ are indexed by $\theta \in \Theta \subset \mathbb{R}$. An agent of type θ believes with certainty that $\mu^{\text{true}} = \mu(\theta)$, where $\mu : \Theta \to \mathbb{R}$ is a weakly increasing function. When $\mu(\cdot)$ is not constant, beliefs vary across residents, and knowing the beliefs of other residents does not lead to any Bayesian updating. This "agree-to-disagree" assumption rules out any inference from prices at $t = 0$. Therefore, for instance, a developer who holds land in equilibrium can realize that it is the most optimistic developer, but this realization fails to change the developer's belief.

As argued by Morris (1996), this heterogeneous prior assumption is most appropriate when investors face an unusual, unexpected situation like the arrival of the shock we are studying. Housing booms historically accompany unanticipated events like the settlement of new cities or the discovery of new resources (Glaeser (2013)). In the case of the U.S. housing boom between 2000

[9] We explore the interaction between speculative and fundamental motives in related work (DeFusco, Nathanson, and Zwick (2016)).

[10] This dispersion is partly due to the limitation that potential residents can buy at most one house. Section IV considers an extension in which potential residents can buy unlimited amounts of housing and rent it to tenants.

and 2006, Mian and Sufi (2009) suggest that the shock was the arrival of new securitization technologies that expanded credit to home buyers, although an equally valid interpretation would be demographic shifts leading to a secular increase in housing demand. The shock to housing demand between 2000 and 2006 is x, and μ^{true} represents the degree to which this shock persists after 2006. Even economists disagreed about μ^{true} during the boom (Gerardi, Foote, and Willen (2010)).

Denote by f_d and f_r the distribution of θ across developers and residents, respectively. We allow these distributions to differ in order to separately study the equilibrium effects of developer and resident beliefs. We make two key assumptions about these distributions:

ASSUMPTION 2: *θ and v are independent for potential residents.*

ASSUMPTION 3: *$\theta_j^{max} \equiv \max \operatorname{supp} f_j$ exists for $j \in \{d, r\}$.*

Assumption 2 guarantees that the two sources of heterogeneity among potential residents at $t = 0$—their beliefs and their housing utility—are independent of one another.[11] Assumption 3 guarantees the existence of a most optimistic developer. Given that developers can access unlimited quantities of financing, an equilibrium would not exist without this regularity condition. We define $\mu_j^{\max} \equiv \mu(\theta_j^{\max})$ for $j \in \{d, r\}$.

To study the marginal effects of disagreement on equilibrium, we adopt the specification

$$\mu(\theta) = \overline{\mu} + z\theta, \tag{19}$$

where $\overline{\mu}, z \geq 0$.[12] When $z = 0$, all agents agree that $\mu^{\text{true}} = \overline{\mu}$. The following assumption guarantees existence in each class of agents of optimists and pessimists relative to the agreement benchmark when $z > 0$:

ASSUMPTION 4: *$\int_{\theta<0} f_j(\theta)d\theta > 0$ and $\int_{\theta>0} f_j(\theta)d\theta > 0$ for $j \in \{d, r\}$.*

Given Assumption 4, larger values of z lead to greater variation in beliefs. The assumption is sufficiently general that an increase to z may also alter the mean beliefs among each group of agents.

D. Land and Housing Market Equilibrium

In an equilibrium, a city is *constrained* if all space is used for housing, *unconstrained* if some space remains as land and the price of land is zero, and *intermediate* if some space remains as land but the price of land is positive. This classification partitions all equilibrium outcomes and will prove useful for describing them.

[11] Although there exist theories of why consumer beliefs may be driven by preferences (Bénabou and Tirole (2016)), independence of beliefs and preferences seems like a reasonable starting point.

[12] The restriction $\overline{\mu} \geq 0$ implies that agents expect a nonnegative growth rate without disagreement and simplifies our analysis by allowing us to focus on the case of a positive shock to the housing market.

The land market clears at t if the sum of $(L_t^{\text{buy}})^*$ across developers equals zero, and the housing market clears at t when the sum of $(H_t^{\text{sell}})^*$ across developers and potential residents equals the sum of $(H_t^{\text{buy}})^*$ across potential residents. The prices p_1^l and p_1^h constitute an equilibrium when the land and housing markets clear at $t = 1$. The following lemma characterizes this equilibrium.

LEMMA 1: *Given N_1, a unique equilibrium at $t = 1$ exists and is given by*

$$p_1^h = \begin{cases} k & \text{if } N_1 < 1 \quad \text{(unconstrained)} \\ kN_1^{1/\epsilon} & \text{if } N_1 \geq 1 \quad \text{(constrained)} \end{cases} \tag{20}$$

and $p_1^l = p_1^h - k$.

Denote the prices in this equilibrium by $p_1^l(N_1)$ and $p_1^h(N_1)$.

These simple expressions for equilibrium prices depend on the model's assumptions in the following way. Because construction is reversible, $p_1^h = p_1^l + k$ and initial conditions such as endowments and the housing stock are irrelevant for prices. Thus, only N_1, the number of arriving potential residents, matters for prices at $t = 1$. If $N_1 < 1$, then the available space S exceeds the number of potential residents who want to buy at $p_1^h = k$. As a result, land is free and $p_1^h = k$. If $N_1 > 1$, then more potential residents want to buy at $p_1^h = k$ than there is available space, so the price of housing rises. In this case, because the elasticity of $D(\cdot)$ for $v \geq k$ is assumed to be ϵ, the pass-through of more potential residents to the house price equals $1/\epsilon$, giving the formula in Lemma 1.

At $t = 0$, an agent of type θ believes with certainty that house and land prices at $t = 1$ will equal $p_1^h(e^{\mu(\theta)x}N_0)$ and $p_1^l(e^{\mu(\theta)x}N_0)$, respectively. Given these beliefs, p_0^l and p_0^h constitute an equilibrium when the land and housing markets clear at $t = 0$. The following lemma characterizes this equilibrium.

LEMMA 2: *Given N_0, x, and z, a unique equilibrium at $t = 0$ exists, and in this equilibrium, $p_0^l = p_0^h - 2k$.*

Denote prices in this equilibrium by $p_0^l(N_0, x, z)$ and $p_0^h(N_0, x, z)$. Sections II and III fully characterize these prices in the cases of agreement and disagreement, respectively.

As above, the result that $p_0^l = p_0^h - 2k$ follows from the reversibility of construction. If $p_0^l < p_0^h - 2k$, then developers would want to buy an infinite amount of land and build houses to sell; if $p_0^l > p_0^h - 2k$, developers would want to buy an infinite amount of housing to revert to land. Markets would not clear in either case, so neither inequality can hold in equilibrium.

II. Equilibrium with Agreement

In this section, we describe the equilibrium house price at $t = 0$ under agreement. This special case of the model, in which $z = 0$, provides the baseline against which we compare the equilibrium under disagreement, in which $z > 0$. Proposition 1 characterizes the equilibrium at $t = 0$ for land holdings, the price of housing, and the effect of the shock x on the price of housing, which we call the *house price boom*.

PROPOSITION 1: *In equilibrium, when $z = 0$ developers hold land at $t = 0$ if and only if $N_0 < 1$. The house price at $t = 0$ equals*

$$p_0^h(N_0, x, 0) = \begin{cases} 2k & \textit{if } N_0 \le e^{-\overline{\mu}x} \quad \textit{(unconstrained)} \\ k + ke^{\overline{\mu}x/\epsilon} N_0^{1/\epsilon} & \textit{if } e^{-\overline{\mu}x} < N_0 < 1 \quad \textit{(intermediate)} \\ k(1 + e^{\overline{\mu}x/\epsilon}) N_0^{1/\epsilon} & \textit{if } N_0 \ge 1 \quad \textit{(constrained)} \end{cases} \tag{21}$$

and the house price boom

$$\frac{p_0^h(N_0, x, 0)}{p_0^h(N_0, 0, 0)} - 1 = \begin{cases} 0 & \textit{if } N_0 \le e^{-\overline{\mu}x} \quad \textit{(unconstrained)} \\ \frac{1}{2}\left(e^{\overline{\mu}x/\epsilon} N_0^{1/\epsilon} - 1\right) & \textit{if } e^{-\overline{\mu}x} < N_0 < 1 \quad \textit{(intermediate)} \\ \frac{1}{2}(e^{\overline{\mu}x/\epsilon} - 1) & \textit{if } N_0 \ge 1 \quad \textit{(constrained)} \end{cases} \tag{22}$$

weakly increases in N_0.

The price at $t = 0$ consists of two terms: one that reflects the housing utility for the marginal buyer today, and one that reflects the common expectation of this marginal utility tomorrow. Using Lemma 1, we may write $p_0^h(N_0, x, 0) = p_1^h(N_0) + p_1^h(e^{\overline{\mu}x} N_0)$. When $N_0 \le e^{-\overline{\mu}x}$, agents expect that $N_1 \le 1$ and $p_1^h = k$, leading to a price today of $2k$. In the intermediate case in which $e^{-\overline{\mu}x} < N_0 < 1$, land is available today but agents agree it will not be tomorrow. When $N_0 \ge 1$, housing is constrained both today and tomorrow.

Under agreement, the house price boom rises monotonically in the level N_0 of demand at $t = 0$. When demand is low, the shock fails to raise prices because agents continue to expect the city to be unconstrained at $t = 1$. Cities at an intermediate level of demand experience intermediate booms, with larger booms in places with more initial demand. Over the range $e^{-\overline{\mu}x} < N_0 < 1$, a larger N_0 indicates that the city is closer to being constrained, so that a greater share of the shock x appears in prices at $t = 1$. When demand is sufficiently high ($N_0 \ge 1$), the shock passes through at a constant rate to the price of housing. Pass-through does not vary with N_0 over this range because of the constant elasticity specification of $D(v)$ in Assumption 1.

III. Equilibrium with Disagreement

This section describes the equilibrium at $t = 0$ under disagreement about future demand growth, which holds when $x, z > 0$ as assumed throughout this section. We study the effect of disagreement on land holdings and the price of housing, the aggregation of beliefs into the price of housing, and the variation in house price booms across cities depending on their initial level of development.

A. Dispersed Homeownership and Land Speculation

Proposition 2 formally describes the equilibrium allocation of land and housing at $t = 0$.

PROPOSITION 2: *In equilibrium, housing is held at $t = 0$ by potential residents of each type $\theta \in \operatorname{supp} f_r$. For land holdings, there exists $N_0^*(x, z) \in \mathbb{R}_{>1} \cup \{\infty\}$ such that:*

- *(Unconstrained) If $N_0 \leq e^{-\mu_d^{max} x}$, then some developers hold land at $t = 0$, and these developers may be of any type $\theta \in \operatorname{supp} f_d$.*
- *(Intermediate) If $e^{-\mu_d^{max} x} < N_0 < N_0^*(x, z)$, then some developers hold land at $t = 0$, and all of these developers have type $\theta = \theta_d^{max}$. Furthermore, there exists $L^* \in (0, S)$ such that if the sum of L_0 across developers for whom $\theta = \theta_d^{max}$ is less than L^*, then the sum of $(L_0^{buy})^*$ across these developers exceeds the sum of $(H_0^{build})^*$ across them.*
- *(Constrained) If $N_0 \geq N_0^*(x, z)$, then no developers hold land at $t = 0$.*

$N_0^*(x, z) = \infty$ if and only if $\int_{\theta \geq \theta_d^{max}} f_r(\theta) d\theta = 0$ and $\int_{\theta < \theta_d^{max}} (e^{\mu_d^{max} x/\epsilon} - e^{\mu(\theta) x/\epsilon})^{-\epsilon} f_r(\theta) d\theta \leq 1$.

In equilibrium, homeownership at $t = 0$ is dispersed among potential residents of all beliefs. A potential resident buys a house at $t = 0$ if $v > p_0^h - \mathrm{E} p_1^h$. The number of homebuyers of type θ equals $N_0 S D(p_0^h(N_0, x, z) - p_1^h(e^{\mu(\theta)x} N_0))$, which is positive for all $\theta \in \Theta$. Positivity depends on Assumption 1, which guarantees that $D(v) > 0$ for any argument. There exist potential residents with arbitrarily high flow utility, so no matter how expensive housing appears to them, some potential residents of each type choose to buy.[13]

Developers choose not to hold housing at the end of $t = 0$ because it is cheaper to hold land and build a house at $t = 1$ for k instead of paying $2k$ at $t = 0$. In the land market, a developer wants to purchase an infinite amount of land if $p_0^l < \mathrm{E} p_1^l$. This situation cannot hold in equilibrium, so for all $\theta \in \operatorname{supp} f_d$,

$$p_0^l(N_0, x, z) \geq p_1^l(e^{\mu(\theta)x} N_0). \tag{23}$$

For θ such that (23) holds with equality, a developer of type θ is indifferent to holding land at $t = 0$. For θ such that (23) is a strict inequality, developers of type θ choose not to hold land at the end of $t = 0$, that is, they either sell it or build houses using the land and sell the houses. As a result, only developers for whom $p_1^l(e^{\mu(\theta)x} N_0) = p_1^l(e^{\mu_d^{\max} x} N_0)$ may hold land at $t = 0$. This is a simple statement of the result that prices in asset markets with disagreement and

[13] This point relates to the work of Cheng, Raina, and Xiong (2014), who find that securitized finance managers did not sell off their personal housing assets during the boom. They interpret this result as evidence that these managers had the same beliefs as the rest of the market about future house prices. An alternative interpretation is that the managers did doubt market valuations, but continued to own housing because they derived sufficiently high utility from housing to compensate for low expected capital gains.

limited short-selling tend to reflect the beliefs of the most optimistic agents (Miller (1977)).

Under two conditions, undeveloped land remains at the end of $t = 0$ and is held only by developers for whom $\theta = \theta_d^{\max}$. First, developers must disagree about their expectations of p_1^l so that $p_1^l(e^{\mu(\theta)x}N_0) < p_1^l(e^{\mu_d^{\max}x}N_0)$ when $\theta < \theta_d^{\max}$. Because $p_1^l(\cdot)$ strictly increases only on $[1, \infty)$, this monotonicity condition holds if and only if $N_0 > e^{-\mu_d^{\max}x}$. The second condition is that some undeveloped land remains at the end of $t = 0$. This condition is met if S exceeds potential resident demand at the price that attracts optimistic developers to hold land:

$$S > N_0 S \int_\Theta D\left(k + p_1^h(e^{\mu_d^{\max}x}N_0) - p_1^h(e^{\mu(\theta)x}N_0)\right) f_r(\theta)d\theta. \tag{24}$$

The proof of Proposition 2 shows that there exists a cutoff $N_0^*(x, z) > 1$ such that the above inequality holds if and only if $N_0 < N_0^*(x, z)$. When $N_0^*(x, z) < \infty$, a sufficiently large number of potential residents will always outbid the most optimistic developers for space.

Proposition 2 shows how the housing and land markets differ in the concentration of ownership among optimists. While potential residents of all beliefs own housing, land is owned only by the most optimistic developers in cities where the initial level of demand takes on intermediate values. The idea that real estate speculation transpires largely in land markets departs from the literature, which focuses mostly on investors in houses.[14]

Developers can carry land over between $t = 0$ and $t = 1$ and thus care about future prices. This feature raises the possibility that they buy land in advance of their immediate construction needs according to their beliefs about future demand. We refer to this behavior as *supply-side speculation*. In the model, if undeveloped land remains and the most optimistic developers own all of it, then they must have bought more than they used for homebuilding (unless they were initially endowed with enough land). Proposition 2 formally states this prediction, which we explore in Section V by examining the balance sheets of U.S. public home builders during the boom and bust of the early 2000s.

B. Belief Aggregation

We next characterize how the equilibrium house price at $t = 0$ aggregates the disparate beliefs of developers and potential residents. For $N_0 \geq N_0^*(x, z)$, we define the aggregate potential resident belief $\mu_r^{\text{agg}}(N_0, x, z)$ to be the unique solution to

$$1 = N_0 \int_\Theta D\left(k(1 + e^{\mu_r^{\text{agg}}x/\epsilon})N_0^{1/\epsilon} - p_1^h(e^{\mu(\theta)x}N_0)\right) f_r(\theta)d\theta, \tag{25}$$

where existence and uniqueness are established in the proof of Lemma 3. This aggregator takes the formula for $p_0^h(N_0, x, 0)$ from Proposition 1 and calculates

[14] See, for example, Barlevy and Fisher (2011), Haughwout et al. (2011), Bayer et al. (2015), and Chinco and Mayer (2015).

the value that must replace $\overline{\mu}$ such that the resulting price clears the market in equilibria in which $z > 0$ and no developers hold land. In this sense, $\mu_r^{\text{agg}}(N_0, x, z)$ describes how the market aggregates the disparate beliefs of potential residents. In particular, $\mu_r^{\text{agg}}(N_0, x, z)$ always lies below the maximal potential resident belief $\mu_r^{\max}$, as shown in the proof of Lemma 3.

The following lemma describes the effect of disagreement on the aggregate potential resident belief under certain conditions.

LEMMA 3: *If* $\int_\Theta \theta f_r(\theta) d\theta = 0$ *and* $\operatorname{supp} f_r \subset [-\overline{\mu}/z, \theta_d^{max}]$*, then* $\mu_r^{agg}(N_0, x, z) > \overline{\mu}$ *and* $\mu_r^{agg}(N_0, x, z) = \overline{\mu} + o(z)$ *as* $z \to 0$.

When the mean potential resident belief is $\overline{\mu}$ and these beliefs fall within certain bounds, the belief aggregator exceeds the mean belief $\overline{\mu}$.[15] The Miller (1977) effect is in force: with disagreement, the aggregate belief implied by market-clearing is greater. However, for small values of z, the Miller (1977) effect is only of second-order importance. To the first order, the decline in demand from pessimists perfectly offsets the increase in demand from optimists, so small disagreement does not change the aggregate belief.

The following proposition characterizes the equilibrium house price under disagreement for all values of N_0:

PROPOSITION 3: *The equilibrium house price at* $t = 0$ *equals*

$$p_0^h(N_0, x, z) = \begin{cases} 2k & \text{if } N_0 \leq e^{-\mu_d^{max} x} \quad \text{(unconstrained)} \\ k + k e^{\mu_d^{max} x/\epsilon} N_0^{1/\epsilon} & \text{if } e^{-\mu_d^{max} x} < N_0 < N_0^*(x, z) \quad \text{(intermediate)} \\ k\left(1 + e^{\mu_r^{agg}(N_0, x, z) x/\epsilon}\right) N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z) \quad \text{(constrained).} \end{cases} \tag{26}$$

Proposition 3 shows how the allocation of land and housing among developers and potential residents affects the price of housing at $t = 0$. There are two important differences between cities with different initial demand levels in terms of how prices aggregate beliefs. First, developer beliefs matter more in intermediate cities, whereas potential resident beliefs matter more in constrained cities. Given the stark assumptions of our model (in particular, the linear production technology for houses as well as the absence of an alternate use for land), prices in intermediate cities reflect only the beliefs of developers. Recent research measures owner-occupant beliefs about the future evolution of house prices.[16] Data on the expectations of home builders would supplement the

[15] The condition on $\operatorname{supp} f_r$ implies that $0 \leq \mu(\theta) \leq \mu_d^{\max}$ for all potential residents, which leads the demand curve for housing to be globally convex with respect to θ. Global convexity implies that disagreement (holding the price constant) stimulates the demand of optimists more than it attenuates the demand of pessimists. Thus, Jensen's inequality implies that the price that clears the market under disagreement exceeds the market-clearing price with agreement, meaning that $\mu_r^{\text{agg}}(N_0, x, z) > \overline{\mu}$.

[16] See Landvoigt (2014), Case, Shiller, and Thompson (2012), Burnside, Eichenbaum, and Rebelo (2015), Soo (2013), Suher (2014), and Cheng, Raina, and Xiong (2014).

research on owner-occupant beliefs to explain house price movements, especially in cities with intermediate levels of development.

The second difference between intermediate-demand and high-demand cities relates to how they aggregate the beliefs of the relevant class of agents. In intermediate cities, prices reflect the most optimistic belief $\mu_d^{\max}$. Other than the maximal value of its support, all other information encoded in the distribution f_d of θ across developers is irrelevant for prices. In contrast, the entire distribution f_r of potential resident beliefs matters for house prices when $N_0 \geq N_0^*(x, z)$. This stark contrast depends on the absence of other constraints on developer size such as risk aversion or capital constraints. The more general point is that when land is held by potential residents, prices need not reflect the most optimistic belief. This is because residents derive utility from housing that may not be correlated with expected capital gains. Moreover, because these utility benefits exhibit diminishing returns, homeownership will tend to be more dispersed than land ownership.

Prices reflect more optimistic beliefs in intermediate cities than in constrained cities when $\mu_d^{\max} > \mu_r^{\text{agg}}(N_0, x, z)$ for all $N_0 \geq N_0^*(x, z)$. The following assumption on the belief distributions is necessary for this relationship to hold.

ASSUMPTION 5: $e^{\mu_r^{max} x/\epsilon} - e^{\mu_d^{max} x/\epsilon} < 1$ *and* $\int_\Theta (1 + e^{\mu_d^{max} x/\epsilon} - e^{\mu(\theta) x/\epsilon})^{-\epsilon} f_r(\theta) d\theta < 1.$

Assumption 5 holds when $f_r = f_d$, so that the distribution of beliefs is the same for each class of agents, but it may hold even if some potential residents are more optimistic than the most optimistic developer. The assumption fails if there is a sufficiently large group of residents with very optimistic beliefs. We invoke Assumption 5 to analyze the effect of disagreement on the house price boom at $t = 0$.

C. The Effect of Disagreement on House Prices

We turn now to the model's key result, which concerns the effect of disagreement on the price of housing at $t = 0$.

PROPOSITION 4: *The effect of disagreement on the house price at $t = 0$ is given by*

$$\frac{p_0^h(N_0, x, z)}{p_0^h(N_0, x, 0)} - 1 = \begin{cases} 0 & \text{if } N_0 \leq e^{-\mu_d^{max} x} \\ \frac{1}{2}\left(e^{\mu_d^{max} x/\epsilon} N_0^{1/\epsilon} - 1\right) & \text{if } e^{-\mu_d^{max} x} < N_0 \leq e^{-\overline{\mu} x} \\ \dfrac{\left(e^{\mu_d^{max} x/\epsilon} - e^{\overline{\mu} x/\epsilon}\right) N_0^{1/\epsilon}}{1 + e^{\overline{\mu} x/\epsilon} N_0^{1/\epsilon}} & \text{if } e^{-\overline{\mu} x} < N_0 \leq 1 \\ \dfrac{\left(e^{\mu_d^{max} x/\epsilon} - e^{\overline{\mu} x/\epsilon}\right) N_0^{1/\epsilon} - \left(N_0^{1/\epsilon} - 1\right)}{(1 + e^{\overline{\mu} x/\epsilon}) N_0^{1/\epsilon}} & \text{if } 1 < N_0 < N_0^*(x, z) \\ \dfrac{e^{\mu_r^{agg}(N_0, x, z) x/\epsilon} - e^{\overline{\mu} x/\epsilon}}{1 + e^{\overline{\mu} x/\epsilon}} & \text{if } N_0 \geq N_0^*(x, z). \end{cases} \tag{27}$$

The increase is positive for $N_0 \in (e^{-\mu_d^{max} x}, 1]$ *and is strictly maximized at* $N_0 = 1$. *If* $\operatorname{supp} f_r \subset [-\overline{\mu}/z, \theta_d^{max}]$ *and* $\int_\Theta \theta f_r(\theta) d\theta = 0$, *then the increase is also positive for all* $N_0 > 1$, *and the marginal effect of small disagreement on the price of housing,*

$$\frac{\partial p_0^h(N_0, x, 0)/\partial z}{p_0^h(N_0, x, 0)} = \begin{cases} 0 & \text{if } N_0 < e^{-\overline{\mu} x} \\ \frac{\theta_d^{max} x}{\epsilon} \frac{e^{\overline{\mu} x/\epsilon} N_0^{1/\epsilon}}{1 + e^{\overline{\mu} x/\epsilon} N_0^{1/\epsilon}} & \text{if } e^{-\overline{\mu} x} \leq N_0 \leq 1 \\ 0 & \text{if } N_0 > 1, \end{cases} \tag{28}$$

is positive only for $e^{-\overline{\mu} x} \leq N_0 \leq 1$.

The first part of Proposition 4 calculates the relative effect of disagreement on $p_0^h(N_0, x, z)$ by comparing the price formulas in Propositions 1 and 2. We now describe the effect of disagreement on the price of housing in each regime using the expressions in Proposition 4.

When $N_0 \leq e^{-\mu_d^{\max} x}$, all developers agree that $p_1^h = k$, because the city will remain unconstrained at $t = 1$. Disagreement between developers on how to interpret the shock x is irrelevant for today's price. Consistent with basic intuition, a fully unconstrained city reacts similarly to an expected shock under agreement and disagreement.

When $e^{-\mu_d^{\max} x} < N_0 \leq e^{-\overline{\mu} x}$, the most optimistic developers expect the city to be constrained, so that $p_1^h > k$, but a developer with the average belief does not. As a result, disagreement raises the price of housing. This increase is larger when N_0 is greater because the price the most optimistic developer expects at $t = 1$ rises with the level of demand.

The analysis of the case $e^{-\overline{\mu} x} < N_0 \leq 1$ is similar, except that now the average developer believes that the city will be constrained in the future. Within this range, $p_1^h > k$ under both the average and most optimistic developer beliefs. The effect of disagreement reflects the extent to which the optimistic developer belief of p_1^h exceeds the average belief, with this difference appearing in the numerator. As N_0 increases, $p_0^h(N_0, x, 0)$ places more weight on beliefs about $t = 1$ relative to the user cost at $t = 0$, so the effect of disagreement on this price increases as well.

The effect of disagreement is most subtle when $1 < N_0 < N_0^*(x, z)$. In this range, disagreement changes the equilibrium from one in which potential residents own all space to one in which the most optimistic developers hold some land. This change alters the house price in two opposing ways, corresponding to the two terms in the numerator in Proposition 4. The first term is positive, as beliefs about p_1^h rise from $ke^{\overline{\mu} x} N_0^{1/\epsilon}$ to $ke^{\mu_d^{\max} x} N_0^{1/\epsilon}$. The second term is negative, as the flow valuation of the marginal buyer at $t = 0$ falls from $kN_0^{1/\epsilon}$ to k. These terms reflect a change in land ownership at the margin: under agreement, the marginal buyer is a potential resident whose housing utility equals $kN_0^{1/\epsilon}$; under disagreement, the marginal buyer is a developer whose flow value of housing is k. The net price effect strictly decreases with N_0 because utility crowd-out increases relative to $N_0^{1/\epsilon}$ as N_0 gets larger.

When $N_0^*(x,z) < \infty$, the effect of disagreement eventually reaches the level given in the final regime of Proposition 4. The effect is positive when $\mu_r^{\text{agg}}(N_0, x, z) > \overline{\mu}$ for all $N_0 \geq N_0^*(x,z)$, which holds given the conditions in the proposition as shown by Lemma 3. When $N_0^*(x,z) = \infty$, for large N_0, the effect of disagreement on the house price approaches $(e^{\mu_d^{\max} x/\epsilon} - e^{\overline{\mu}x/\epsilon} - 1)/(1 + e^{\overline{\mu}x/\epsilon})$, which the proof of Proposition 4 shows is positive.

The key result is that the effect of disagreement in the $N_0 \geq N_0^*(x,z)$ regime is less than that at $N_0 = 1$. This comparison depends on Assumption 5, which guarantees that $\mu_r^{\text{agg}}(N_0, x, z) < \mu_d^{\max}$, and on Assumption 2, which leads to dispersion of homeownership among potential residents of all beliefs.

The last part of Proposition 4 isolates the effect of disagreement by studying a small increase in z from $z = 0$. As shown by Lemma 3, this increase has no first-order effect on $\mu_r^{\text{agg}}(N_0, x, z)$ (under the given conditions on f_r). In contrast, the increase in z raises the maximal developer belief $\mu_d^{\max}$ to the first order because $\partial \mu_d^{\max}/\partial z = \theta_d^{\max} > 0$. As a result, the marginal effect of small disagreement is positive only in the intermediate region of N_0.

In summary, disagreement raises the price of housing everywhere except cities where the level of demand is very low and possibly cities where the level of demand is very high but many extreme optimists and pessimists exist. For specifications of the joint distribution of resident and developer beliefs that satisfy Assumption 5 (such as identical distributions in the two subpopulations), disagreement raises the price most in cities with an intermediate level of development.

D. Variation in Price Booms across Cities

Proposition 5 characterizes how the house price boom under disagreement varies across cities.

PROPOSITION 5: *The house price boom under disagreement,*

$$\frac{p_0^h(N_0, x, z)}{p_0^h(N_0, 0, z)} - 1 = \begin{cases} 0 & \text{if } N_0 \leq e^{-\mu_d^{max} x} \\ \frac{1}{2}\left(e^{\mu_d^{max} x/\epsilon} N_0^{1/\epsilon} - 1\right) & \text{if } e^{-\mu_d^{max} x} < N_0 \leq 1 \\ \frac{1}{2}\left(e^{\mu_d^{max} x/\epsilon} - 2 + N_0^{-1/\epsilon}\right) & \text{if } 1 < N_0 < N_0^*(x,z) \\ \frac{1}{2}\left(e^{\mu_r^{agg}(N_0,x,z)x/\epsilon} - 1\right) & \text{if } N_0 \geq N_0^*(x,z), \end{cases} \tag{29}$$

is strictly maximized at $N_0 = 1$.

As in Proposition 1, we define the price boom as the effect of the shock x on $p_0^h(N_0, 0, z)$. This boom can be decomposed into the product of $p_0^h(N_0, x, 0)/p_0^h(N_0, 0, 0)$ (the marginal effect of x when $z = 0$ given by Proposition 1) and $p_0^h(N_0, x, z)/p_0^h(N_0, x, 0)$ (the marginal effect of z given by Proposition 4).[17] The former monotonically increases in the level of initial

[17] Here, we are using the fact that $p_0^h(N_0, 0, z) = p_0^h(N_0, 0, 0)$ because z becomes irrelevant when $x = 0$.

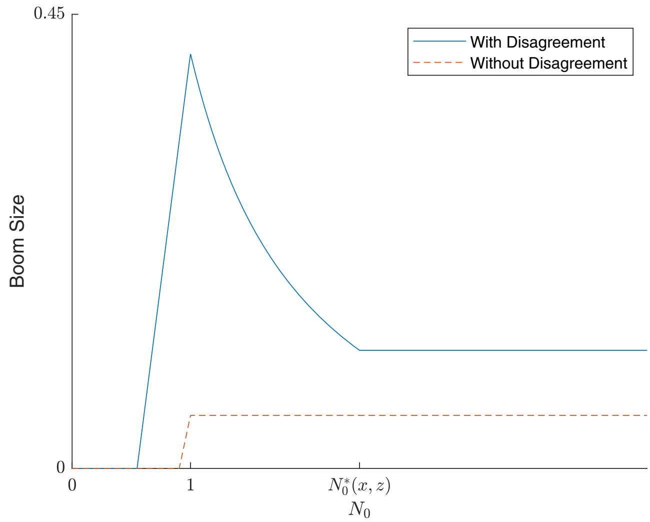

Figure 2. House price boom for different initial demands. The boom size equals the relative size of the equilibrium house price at $t = 0$ with versus without the shock x, that is, $p_0^h(N_0, x, z)/p_0^h(N_0, 0, z) - 1$ with disagreement and $p_0^h(N_0, x, 0)/p_0^h(N_0, 0, 0) - 1$ without disagreement, where N_0 equals the number of potential residents at $t = 0$ relative to the amount of space in the city, and z equals the level of disagreement at $t = 0$. The parameter values used to generate this figure are $x = 0.5$, $z = 1$, $\epsilon = 1$, $\overline{\mu} = 0.2$, and $f_r = f_d$, with 90% of agents having $\theta = -1/9$ and 10% having $\theta = 1$. These parameters are defined in Section I. (Color figure can be viewed at wileyonlinelibrary.com)

demand N_0, whereas the latter strictly peaks at $N_0 = 1$. Proposition 5 shows that the combined effect also strictly peaks at $N_0 = 1$, meaning that, with disagreement, the result that demand shocks raise prices most in constrained cities no longer holds.

The intuition behind Proposition 5 is similar to that of Proposition 4. Cities with low initial demand experience no price boom because all developers agree that they will remain unconstrained at $t = 0$. For intermediate cities where $e^{-\mu_d^{\max} x} < N_0 \leq 1$, prices rise according to the beliefs of the most optimistic developer. For intermediate cities with $1 < N_0 < N_0^*(x, z)$, prices rise less when $N_0 > 1$ because of the utility crowd-out of homeowners by developers. Finally, prices rise for $N_0 \geq N_0^*(x, z)$ according to the aggregate beliefs of all potential residents. Under Assumption 5, this aggregate belief falls short of the most optimistic developer belief, so the price boom is largest when $N_0 = 1$.

To illustrate the variation in the price boom across cities, Figure 2 plots the expression from Proposition 5 across different values of N_0, both for a positive value of z and for $z = 0$. We set $f_r = f_d$ so that the conditions of Assumption 5 hold. As can be seen in the figure, disagreement amplifies the boom everywhere except in cities with small initial demand where disagreement has no effect. The amplification is largest in cities with intermediate values of initial demand, leading the boom to be largest in the case of disagreement at $N_0 = 1$.

The boom in the case of agreement rises monotonically with respect to the level of initial demand.

E. Disagreement and Welfare

Disagreement can reallocate space from pessimists to optimists. This reallocation destroys welfare if some of the pessimists are potential residents with high flow utility and some of the optimists are developers or potential residents with lower flow utility. To formally analyze the effect of disagreement on welfare, we adopt the "belief-neutral Pareto efficiency" criterion proposed by Brunnermeier, Simsek, and Xiong (2014) as a welfare measure for models with heterogeneous beliefs. An allocation is belief-neutral Pareto-efficient if it is Pareto-efficient under all linear combinations of agent beliefs. Proposition 6 shows that disagreement reduces welfare in intermediate and constrained cities.

PROPOSITION 6: *The equilibrium allocation is belief-neutral Pareto-efficient if $z = 0$ or $N_0 \leq e^{-\mu_d^{max} x}$ and is belief-neutral Pareto-inefficient otherwise.*

While disagreement may lead only to welfare-neutral transfers in the stock market, this result demonstrates that disagreement tends to lower welfare in the housing market.

The reallocations that improve welfare when $z > 0$ are as follows. When $e^{-\mu_d^{\max} x} < N_0 < N_0^*(x, z)$, there exists a potential resident who chooses not to buy despite having flow utility $v > k$. The resource cost of building a house at $t = 0$ instead of $t = 1$ is k, so there exists a cash transfer from this potential resident to a developer that makes them both better off if the developer builds a house and gives it to the potential resident. For $N_0 \geq N_0^*(x, z)$, there exist potential residents with flow utilities v^1 and v^2 such that $v^1 < v^2$ and the potential resident with $v = v^1$ buys, whereas the one with $v = v^2$ does not. With a suitable cash transfer, changing which potential resident owns the house improves the welfare of both. Under the $z = 0$ equilibrium allocation, these situations never occur.

IV. Extensions and Additional Predictions

A. Equity Financing

The developers in the baseline model raise any needed funds at $t = 0$ using debt. Section I of the Internet Appendix presents an extension in which developers may raise funds only through equity offerings. The analysis formalizes results that we explore empirically in Section V, in which we examine the market value and short-selling of the equity of public developers during the 2000 to 2006 U.S. housing boom.

In this extension, *equity investors* constitute a third class of agents. Across equity investors, the distribution of beliefs f_i about μ^{true} satisfies Assumption 3, which guarantees the existence of a most optimistic equity investor, and

Assumption 4, which ensures disagreement when $z > 0$. Some of the developers endowed with land at $t = 0$ may raise funds by selling claims on their $t = 1$ liquidation values to these investors.[18] Each equity investor may borrow freely at a rate of zero to finance positive purchases of these claims. In contrast, to sell these claims short, equity investors must pay a positive proportional fee. Furthermore, each equity investor may sell short a limited number of claims. In equilibrium, a price exists for the claim on each developer able to access the equity market such that the value of the claims sold by the developer equals the net quantity demanded by equity investors.

The following proposition characterizes the price of housing, the allocation of land holdings, the price of developer equity, and the total short position of equity investors at $t = 0$.[19] We define $p_0^h(N_0, x, z, f_r, f_d)$ to be the equilibrium value of p_0^h in Proposition 2 given the potential resident belief distribution f_r and the developer belief distribution f_d, and we denote by $\theta_i^{\max}$ and $\mu_i^{\max} = \mu(\theta_i^{\max})$ the type and belief of the most optimistic equity investor.

PROPOSITION 7: *If $xz = 0$, then the aggregate value of short claims equals zero, and there exists an equilibrium in which no equity issuance nor land purchases occur. If $xz > 0$,*

- *if $\sum_{\theta > \theta_i^{max}} L_0 = 0$, then the equilibrium house price equals $p_0^h(N_0, x, z, f_r, f_i)$;*
- *if $\sum_{\theta < \theta_i^{max}} \frac{L_0}{S} > e^{-\mu_i^{max} x}$, among developers without access to equity then there exists N_0 for which the following hold:*

  (a) some developers issue equity with positive value,
  (b) $\sum (L_0^{buy})^* > \sum (H_0^{build})^*$ across developers issuing equity,
  (c) the total short position in this equity is positive for some values of the short fee,
  (d) the equity price for each such developer exceeds the price under $x = 0$, and
  (e) the equity price for each such developer falls from $t = 0$ to 1 iff $\mu_i^{max} > \mu^{true}$.

If none of the developers endowed with land are more optimistic than the most optimistic equity investor (for example, if the developer and investor belief distributions coincide), the pricing formula in Proposition 2 carries over to the model with equity financing with one difference: now the most optimistic equity investor belief replaces the most optimistic developer belief. The non-monotonicity of the house price boom and the disagreement price effect also

[18] The developers who cannot access the equity market represent small firms and nonprofit landowning entities like governments and Native American tribes that are not able to issue equity.

[19] In this extension, land that remains undeveloped at the end of $t = 0$ pays a small positive dividend at the beginning of $t = 1$. Proposition 7 specifies the limiting equilibrium price as this dividend goes to zero. The sole purpose of this dividend is to ensure the existence of equilibrium when $p_0^h(N_0, x, z) = 2k$.

carry over as long as $\theta \leq \theta_i^{\max}$ for nonlandowning developers.[20] The most optimistic investor prices all developer equity because long positions are unlimited, while short positions are costly. When $\sum_{\theta > \theta_i^{\max}} L_0 = 0$, all developers are willing to sell equity backed by their landholdings to investors or to sell their landholdings at the optimistic investor valuation. As a result, the equilibrium land price coincides with the optimistic investor valuation, leading to an equilibrium house price of $p_0^h(N_0, x, z, f_r, f_i)$.[21]

Proposition 7 also characterizes quantities in the equity and land markets without and with disagreement. Without disagreement, short-selling never occurs because all investors agree on the equity valuations and the short fee is positive. Equity issuance and land purchases by developers are not guaranteed to occur because developers are indifferent between selling land, selling equity, and holding land until $t = 1$.

With disagreement, equity issuance, land purchases by equity-issuing developers, and short-selling of that equity all occur in equilibrium as long as the short fee is small enough and pessimistic developers without equity market access hold enough land. With enough pessimistic developers, other developers must raise funds from optimistic investors to buy out the pessimists and satisfy the expected demand of potential residents. Equity-issuing developers buy land in excess of their immediate construction needs, resembling the optimistic developers characterized by Proposition 2.

B. Housing Rental Market

In the baseline model, potential residents derive housing utility only from owning and may own only one house. To explore the importance of these restrictions, Section II of the Internet Appendix presents an extension in which rental contracts are available and potential residents may operate as landlords.

In this extension, potential residents may buy any positive amount of housing. They choose how much housing to lease as landlords and how much to keep as owner-occupied housing. Potential residents may also choose to rent housing as tenants. A fraction χ of potential residents receive housing utility v if and only if they occupy at least one unit of housing as a tenant, and the remaining potential residents receive v if and only if they occupy at least one unit of housing as an owner-occupant. Rental prices at $t = 0$ and $t = 1$ clear the market, so that the quantity of housing chosen to be leased by landlords equals the quantity chosen to be rented by tenants.

[20] In this case, $\theta_d^{\max} \leq \theta_i^{\max}$, so Assumption 5 (on which Propositions 4 and 5 rely) holds with $\mu_i^{\max}$ in place of $\mu_d^{\max}$.

[21] Section I of the Internet Appendix analyzes in detail the case in which $\sum_{\theta > \theta_i^{\max}} L_0 > 0$. In this case, some landowning developers are more optimistic than the most optimistic investor. There may exist values of N_0 at which these optimistic developers hold land in equilibrium, with the $t = 0$ house price independent of the beliefs and endowments of all other developers.

We define $N_0^*(x, z, f_r)$ to be the value of N_0^* in Proposition 2 given the potential resident belief distribution f_r, and we define the distribution f_r^χ by $f_r^\chi(\theta) = \chi \mathbf{1}_{\theta_r^{\max}} + (1-\chi) f_r(\theta)$. The following proposition describes the $t = 0$ equilibrium in the rental extension.

PROPOSITION 8: *If $xz = 0$, χ equals the share of the housing stock that is rented. If $xz > 0$,*

- *the equilibrium house price equals $p_0^h(N_0, x, z, f_r^\chi, f_d)$;*
- *there exists $\chi^*(x, z) \in (0, 1]$ such that the house price boom is strictly maximized at $N_0 = 1$ if $\mu_r^{max} < \mu_d^{max}$ or if $\mu_r^{max} \geq \mu_d^{max}$ and $\chi < \chi^*(x, z)$; and*
- *the house price boom depends on χ only if $N_0 \geq N_0^*(x, z, f_r^\chi)$, in which case it increases.*

As shown by Proposition 8, the equilibrium house price assigns additional weight to the optimistic resident belief in proportion to the underlying share of potential residents who prefer renting. This skewing occurs because only the most optimistic potential residents become landlords in equilibrium.

Proposition 8 further shows that the nonmonotonicity of the house price boom characterized by Proposition 5 carries over to this setting under certain conditions. If $\mu_r^{\max} < \mu_d^{\max}$, then the house price boom remains maximized at $N_0 = 1$ for all χ because the developers pricing housing at that point are more optimistic than the residents pricing housing in markets where developers do not participate. In the more interesting case in which $\mu_r^{\max} \geq \mu_d^{\max}$, the price boom is maximized at $N_0 = 1$ only for sufficiently small values of χ. This $\chi < \chi^*(x, z)$ constraint need not be very restrictive—if $\mu_r^{\max} = \mu_d^{\max}$, then $\chi^*(x, z) = 1$, so the boom remains nonmonotonic as long as some positive measure of potential residents prefer owner-occupancy to renting.

The last part of Proposition 8 delivers the empirical prediction that among identical cities where no land remains, cities in which a higher share of housing is rented without disagreement experience larger house price booms with disagreement. We explore this prediction in Section V.

C. Continuous Housing Supply Elasticity

A key statistic used to analyze house price booms is the elasticity of housing supply (Glaeser, Gyourko, and Saiz (2008), Mian and Sufi (2009)). In the baseline model, this elasticity is zero when all land is used for housing and infinite when some undeveloped land remains. Disagreement amplifies the house price boom most when $e^{-\mu_d^{\max} x} < N_0 < N_0^*(x, z)$, the parameter region in which supply is perfectly elastic at $t = 0$ but is expected by the optimistic developers to be perfectly inelastic at $t = 1$. This characterization suggests that disagreement amplifies price booms most when supply is elastic today but expected to be inelastic tomorrow. To see how robust this conclusion is, Section III of the Internet Appendix extends the rental market model to the case in which the supply elasticity declines continuously with the level of initial demand.

In this extension, developers may rent out undeveloped land on spot markets each period to firms that use the city's land as an input. The land demand from these firms at $t = 0$ and $t = 1$ is given by a continuously differentiable, decreasing, positive function of the spot rental rate. The limiting values of this function for small and large rents are sufficiently extreme that a unique equilibrium exists, and this function becomes weakly more inelastic for larger rents.

Given a house price p_t^h, a unique partial equilibrium exists in which developers optimize and the land markets clear. We define the elasticity of supply to be the partial derivative of the log of the housing stock chosen by developers with respect to p_t^h, all normalized by the house rent r_t^h.[22] As shown in Section III of the Internet Appendix, there exists a continuous, decreasing function $\epsilon^s : \mathbb{R}_+ \to \mathbb{R}_+$ such that the supply elasticity equals $\epsilon^s(N_t)$ when $z = 0$.

It is difficult to provide an exact solution for the house price boom (a counterpart for Proposition 5) in this setting. Instead, the following proposition solves for the marginal impact of the shock x on the equilibrium house price $p_0^h(N_0, x, z, \chi)$ when x or z is zero. We then use the formulas from these special cases to describe the distinct roles played by the current and future housing supply elasticity.

PROPOSITION 9: *The following approximation holds exactly for $x = 0$ or $z = 0$:*

$$\frac{\partial \log p_0^h(N_0, x, z, \chi)}{\partial x} \approx \left(\overline{\mu} + \frac{\epsilon^s(N_0)\theta_d^{max} + \chi\epsilon\theta_r^{max} + (1-\chi)\epsilon\theta_r^{avg}}{\epsilon^s(N_0) + \epsilon} z\right) \frac{\left(\epsilon^s\left(e^{\overline{\mu}x}N_0\right) + \epsilon\right)^{-1}}{1 + e^{-\int_0^x \left(\epsilon^s\left(e^{\overline{\mu}x'}N_0\right)+\epsilon\right)^{-1}\overline{\mu}dx'}}, \tag{30}$$

where $\theta_r^{avg} = \int_\Theta \theta f_r(\theta)d\theta$.

Proposition 9 confirms the intuition from the main model on the distinct roles played by the supply elasticities at $t = 0$ and $t = 1$. The fraction multiplying z aggregates beliefs across market participants. When the current elasticity $\epsilon^s(N_0)$ is higher, more weight is placed on the optimistic developer type $\theta_d^{\max}$ because these developers constitute a larger share of the marginal buyers in the market. A larger $\epsilon^s(N_0)$ implies a greater influence of disagreement on the house price today. Similarly, a higher χ increases the share of marginal buyers who are landlords and raises the weight on the optimistic resident type $\theta_r^{\max}$.[23]

[22] Normalizing by r_t^h instead of p_t^h allows us to use the same function to analyze the supply elasticity at $t = 0$ and $t = 1$. Given the finite horizon of the model, the price at $t = 0$ is roughly double the price at $t = 1$, so supply would be about twice as elastic at $t = 0$ if we normalized by p_t^h. A unique equilibrium rent exists for $\chi > 0$; when $\chi = 0$, we choose the rent that obtains as the limit as $\chi \to 0$, which is a valid equilibrium rent for $\chi = 0$.

[23] The beliefs of nonlandlord potential residents appear only through θ_r^{avg}, and hence their disagreements relative to this average are irrelevant for this formula. The intuition for this result is similar to the intuition for the limit result in Lemma 3. When x is near zero, the variation in

In contrast, a greater value of $\epsilon^s(e^{\overline{\mu}x}N_0)$—the supply elasticity at $t = 1$ under the mean belief—lowers the house price boom at $t = 0$. A larger supply elasticity at $t = 1$ implies smaller pass-through of the shock x to the price at $t = 1$. This pass-through is given by $(\epsilon^s(e^{\overline{\mu}x}N_0) + \epsilon)^{-1}$. Similarly, a greater value of $\epsilon^s(e^{\overline{\mu}x}N_0)$ lowers the value of the integral, which captures the expected price tomorrow relative to today. When this ratio is lower, the pass-through of the shock x to the price at $t = 0$ is lower.

In summary, the price boom is largest when $\epsilon^s(N_0)$ is large but $\epsilon^s(e^{\overline{\mu}x}N_0)$ is small. This combination occurs when the shock x pushes the city from having a high supply elasticity in the present to possibly having a low supply elasticity in the future. The possibility of such a transition amplifies the role of disagreement relative to cases in which transition to low supply elasticity is unlikely or has already occurred. Thus, the intuition from the baseline model that disagreement affects the size of the house price boom most for cities at an intermediate level of development translates to this more general setting.

V. Stylized Facts of the U.S. Housing Boom and Bust

This section uses data from the U.S. housing boom between 2000 and 2006 to provide evidence consistent with the model's predictions about the house price boom at $t = 0$. We first document the importance of the relationship between the price of raw land and the price of housing across cities, which supports the model's focus on how disagreement interacts with potential land constraints. We then describe the speculative behavior in land markets among public homebuilders, who resemble the optimistic developers in the model extension in which developers issue equity. Finally, we show how the model can be used to understand the booms in the cross section of U.S. cities, as well as across neighborhoods within cities.

A. The Central Importance of Land Prices

A key assumption of the model is that housing supply is limited in the long run by development constraints. These constraints lead land prices to rise during a housing boom, as developers anticipate the exhaustion of land. As a result, house prices and land prices rise in unison as shown by the result in Lemma 2 that $p_0^h = p_0^l + 2k$ in equilibrium.

Tracing house price increases to land prices distinguishes our model from "time-to-build." Traditionally, housing supply has been modeled as inelastic in the short run and elastic in the long run (DiPasquale and Wheaton (1994), Mayer and Somerville (2000)). This paradigm described the U.S. housing market very well for a time. Topel and Rosen (1988) show that, essentially, all

beliefs about p_1^h among potential residents is small for any z. This small disagreement does not alter potential resident housing demand because the decline in demand from pessimists perfectly offsets the increase in demand from optimists.

variation in house prices between 1963 and 1983 in the United States came from changes to the construction cost of structures. Temporary shortages of inputs needed to build a house, such as drywall and skilled labor, could explain this pattern, with the fluctuations in these input prices causing house price cycles.

Between 1983 and 2000, a secular shift occurred in housing supply in the United States Land prices became a much larger share of house prices (Davis and Heathcote (2007)), especially in certain cities (Davis and Palumbo (2008)). A large literature, surveyed by Gyourko (2009), attributes this change to the rise of government regulations restricting housing supply. These rules bound city growth by limiting the number of building permits issued to developers. When demand to live in the city rises, land prices increase because the city cannot expand.

Developers in a city without supply restrictions today might expect them to arrive in the future. Anticipating these regulatory changes, developers bid up land prices immediately after a demand shock, even in the absence of current building restrictions. In such a city, supply is elastic in the short run, but inelastic in the long run—it's "arrested development." In the baseline model, cities at an intermediate development level exhibit an extreme case of arrested development, with infinite supply elasticity at $t = 0$ and zero supply elasticity at $t = 1$ if all land is developed. The equilibrium presented in Proposition 9 considers a more general case in which the supply elasticity declines continuously with the level of initial demand.

Under arrested development, a nationwide housing demand shock can increase land prices *everywhere*, not just in cities where regulations currently restrict supply. Land prices rise even in areas with rapid construction. In contrast, time-to-build predicts construction cost increases and not land price increases. If temporary input shortages are driving house prices, then land prices, which are fully forward looking, should remain flat.

To assess the relative importance of land prices, we gather data on land prices and construction costs at the city level between 2000 and 2006. We use land prices measured directly from parcel transactions during this time. This approach contrasts with that used by Davis and Heathcote (2007) and Davis and Palumbo (2008), who measure land prices as the residual when construction costs are subtracted from house prices. A direct measure of land prices addresses concerns that such residuals capture something other than land prices between 2000 and 2006. The land price data we use are the indices constructed by Nichols, Oliner, and Mulhall (2013). Using land transaction data, they regress prices on parcel characteristics and then construct city-level indices from the coefficients on city-specific time dummies.

We measure construction costs using the R.S. Means construction cost survey. This survey asks home builders in each city to report the marginal cost of building a square foot of housing, including all labor and materials costs. Survey responses reflect real differences across cities. In 2000, the lowest cost is \$54 per square foot and the highest is \$95; the mean is \$67 per square foot and the standard deviation is \$9. This survey has been used to study time series and spatial variation in residential construction costs (Glaeser and Gyourko (2005), Gyourko and Saiz (2006), Gyourko (2009)).

As shown by Lemmas 1 and 2, the assumptions of our model imply that house prices must equal land prices plus construction costs: $p_t^h = p_t^l + k_t$. Log-differencing this equation between 2000 and 2006 yields

$$\Delta \log p^h = \alpha \Delta \log p^l + (1 - \alpha)\Delta \log k, \tag{31}$$

where Δ denotes the difference between 2000 and 2006 and α is land's share of house prices in 2000. The factor that matters more should vary more closely with house prices across cities. Because α and $1 - \alpha$ are less than 1, the critical factor should also rise more than house prices do.

Figure 3 plots for each city the real growth in construction costs and land prices between 2000 and 2006 against the corresponding growth in house prices. Construction costs did rise during this period, but they rose substantially less than land prices. The growth in construction costs displays very little variation across cities. The time-to-build hypothesis then does explain some of the level of house price increases in the United States during the boom, but none of the cross-sectional variation. Land prices display the opposite pattern, rising substantially and exhibiting a high correlation with house prices. Each city's land price increase also exceeds its house price increase. This evidence underscores the central importance of land prices for understanding the cross section of the house price boom, and broadly supports the relative contribution of arrested development over time-to-build.

B. Supply-Side Speculation by Home Builders

Proposition 2 predicts that, as long as they are not endowed with too much land, optimistic developers amass land beyond their immediate construction needs at $t = 0$ in intermediate cities. Proposition 7 extends this result to the case in which developers finance themselves with equity and offers additional predictions about the developer equity market. We examine these predictions among a class of developers for whom rich data are publicly available: public home builders. We focus on the eight largest firms and hand-collect landholding data from their annual financial statements between 2001 and 2010.[24]

The eight equity-financed large firms we study nearly tripled their landholdings between 2001 and 2005, as shown in Figure 4, Panel A.[25] Consistent with Propositions 2, 7(a), and 7(b), these land acquisitions far exceed land needed for new construction. Annual home sales increased by 120,000 homes

[24] Our analysis complements Haughwout et al. (2012), who empirically study the home building industry and present similar facts based on different data.

[25] During this period, these firms bought back \$400 million of equity on net, equal to approximately 0.7% of total assets in 2005. Negative equity issuance is inconsistent with the static model in Section I of the Internet Appendix, in which developers have equity outstanding at $t = 0$ only if they issue equity at $t = 0$. Negative equity issuance may be consistent with a dynamic model in which developers can rely on retained earnings to finance additional land purchases or can issue debt in addition to equity.

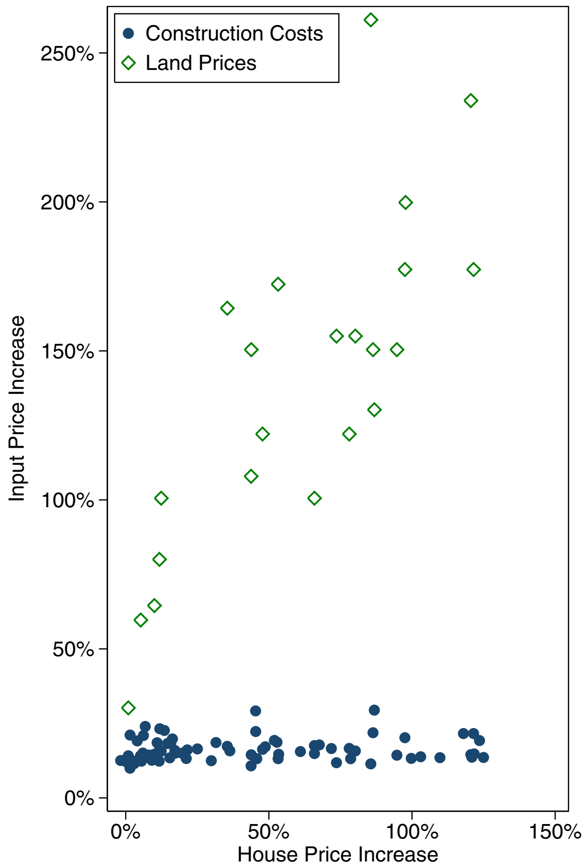

Figure 3. Input price and house price increases across cities, 2000–2006. We measure construction costs for each city using the R.S. Means survey figures for the marginal cost of a square foot of an average quality home, deflated by the consumer price index for all urban consumers (CPI-U). Gyourko and Saiz (2006) provide further information on the survey. Land price changes come from the hedonic indices calculated in Nichols, Oliner, and Mulhall (2013) using land parcel transactions, and house prices come from the second-quarter FHFA housing price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data. For land prices, we have data for Atlanta, Baltimore, Boston, Chicago, Dallas, Denver, Detroit, Houston, Las Vegas, Los Angeles, Miami, New York, Orlando, Philadelphia, Phoenix, Portland, Sacramento, San Diego, San Francisco, Seattle, Tampa, and Washington D.C. (Color figure can be viewed at wileyonlinelibrary.com)

between 2001 and 2005, while landholdings increased by 1,100,000 lots. One lot can produce one house, so landholdings rose more than *nine times* relative to home sales. In 2005, Pulte changed the description of its business in its 10-K to “We consider land acquisition one of our core competencies.” This language appeared until 2008, when it was replaced by, “Homebuilding operations represent our core business.”

Having amassed large land portfolios, these firms subsequently suffered significant capital losses. Figure 4, Panel B, documents the dramatic rise

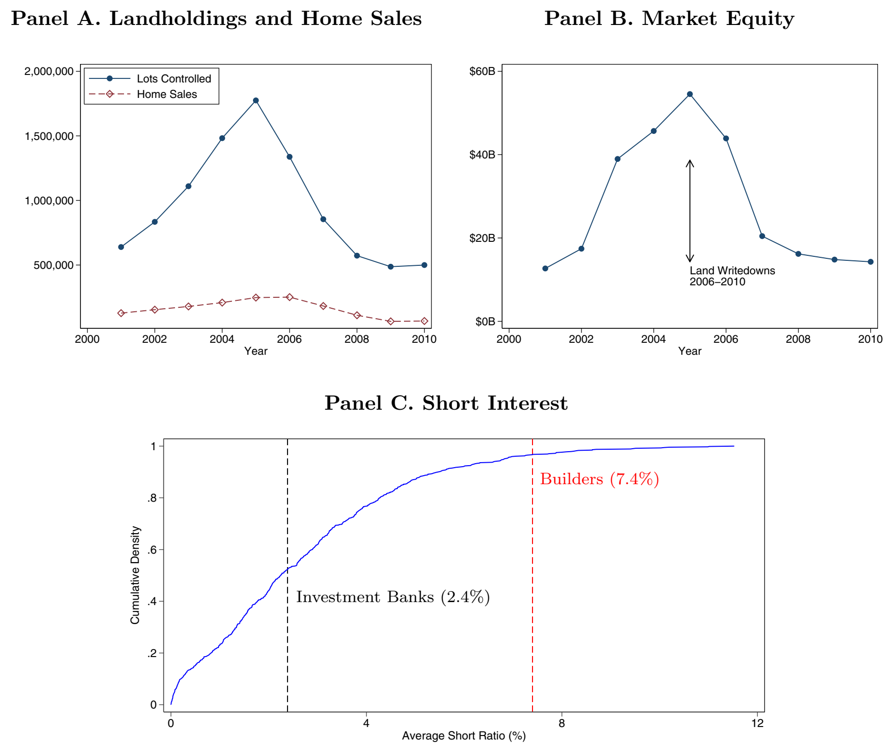

Figure 4. Supply-side speculation among U.S. public home builders, 2001–2010. Panels A and B: Data come from the 10-K filings of Centex, Pulte, Lennar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific, the eight largest public U.S. home builders in 2001. "Lots Controlled" equals the sum of lots directly owned and those controlled by option contracts. The cumulative writedowns to landholdings between 2006 and 2010 among these home builders total $24 billion. Panel C: Short interest is computed as the ratio of shares currently sold short to total shares outstanding. Monthly data series for shares short come from Compustat and for shares outstanding come from CRSP. We compute mean short interest between 2000 and 2006 for each six-digit NAICS industry and plot the cumulative distribution of these means. Builder stocks are classified as those with NAICS code 236117 and investment bank stocks are those with NAICS code 523110. (Color figure can be viewed at wileyonlinelibrary.com)

and fall in the total market equity of these home builders between 2001 and 2010. Home builder stocks rose 430% and then fell 74% over this period. By Proposition 7(d), the rise is consistent with a positive shock $x > 0$ at $t = 0$. If we interpret the period between 2006 and 2010 as that between $t = 0$ and $t = 1$, then, by Proposition 7(e), the losses are consistent with a realization of μ^{true} below the optimistic investor belief $\mu_i^{\max}$.

The majority of the losses borne by home builders arose from losses on the land portfolios they accumulated from 2001 to 2005. In 2006, these firms began reporting write-downs to their land portfolios. At $24 billion, the value of the land losses between 2006 and 2010 accounts for 61% of the market equity losses over this time period.[26] The home builders bore the entirety of their land portfolio losses. The absence of a hedge against downside risk supports the theory that home builder land acquisitions represented optimistic beliefs.

It is hard to argue that this rise and fall in equity prices reflect any monopoly rents home builders earned by building houses during this period. During the boom, home building was extremely competitive. Haughwout et al. (2012) document that the largest 10 home builders had less than a 30% market share throughout the boom, with firms outside the largest 60 constituting over half of market share. Although some consolidation occurred between 2000 and 2006, these numbers portray an extremely competitive market. If anything, consolidation may reflect purchases by optimistic firms of pessimists who chose to abstain from land speculation.

Consistent with Prediction 7(c), these home builders witnessed heightened short-selling of their equity during the boom. Figure 4, Panel C, plots the distribution of the average monthly short interest ratio, defined as the ratio of shares currently sold short to total shares outstanding, across all industries between 2000 and 2006. The short interest of home builder stocks lies in the 95$^{\text{th}}$ percentile, meaning that investors short-sold this industry more than nearly all others during the boom. As a point of comparison, the short interest in home builders was triple that in investment banks, another industry exposed to housing at this time. The short interest in home builders provides direct evidence of disagreement over the value of their land portfolios.[27]

Several recent papers argue that optimism about house prices was widespread between 2000 and 2006. For instance, Foote, Gerardi, and Willen (2012) document 12 facts about the mortgage market during this time that are inconsistent with incentive problems between borrowers and lenders, but consistent with beliefs of borrowers and lenders that house prices would continue to rise. Case, Shiller, and Thompson (2012) directly survey homeowners during the boom and find that they expected continued appreciation in house prices over the next decade, as opposed to the bust that eventually occurred. Cheng, Raina, and Xiong (2014) find that securitized finance managers did not sell off their personal housing assets during the boom, indicating that these managers had similarly optimistic beliefs relative to the rest of the market. The disagreement our model relies on is consistent with such widespread optimism. Homeowners and investors can be optimistic on average, with dispersion in beliefs around this optimistic mean. Furthermore, only the *most* optimistic investors price land in the model. Thus, a few extraordinarily

[26] Total impairments during this time period, which include impairments to goodwill and joint ventures as well as unclassified impairments, equal $29 billion.

[27] An earlier draft of this paper provided the time series of short interest in home builder stock from 2001 to 2010. Short interest rose from 2001 to 2006, but rose even further from 2006 to 2009. Home builder short interest was highest as the bust was beginning. This peak may indicate that disagreement reached its peak after the boom, complicating the idea that disagreement was high during the boom. Alternatively, the late peak could indicate that shorting is more attractive for pessimists when they anticipate a bust in the near future.

optimistic investors have a large price impact, even when nearly all people agree about the future of house prices.

C. The Cross Section of Cities

House price increases differed markedly across cities during the 2000 to 2006 U.S. housing boom. Propositions 1 and 5 derive house price increases as a function of city development levels and disagreement. We test these predictions by interpreting them as comparative statics and then examining them against the empirical variation in house price increases across cities.

Davidoff (2013) analyzes the ability of housing supply elasticities to explain the cross-city variation in house price booms during this time. Part of his analysis measures supply elasticities using price and quantity growth during the 1980s. He finds that many of the largest bubbles (annualized 2000 to 2007 boom minus annualized 2007 to 2010 bust) occurred in cities with historically elastic supply and that many of these cities are in the "sand states" of Arizona, California, Florida, and Nevada.[28] We present findings similar to these and then interpret them using our theoretical results.

We use the Federal Housing Finance Agency's metropolitan statistical area quarterly house price indices, and we measure the housing stock in each city at an annual frequency by interpolating the U.S. Census's decadal housing stock estimates with its annual housing permit figures. Throughout, we focus on the 115 metropolitan areas for which the population in 2000 exceeds 500,000. The boom consists of the period between 2000 and 2006.

Figure 5, Panel A, plots construction and house price increases across cities. The house price increases vary enormously across cities, ranging from 0% to 125% over this brief six-year period. The largest price increases occurred in two groups of cities. The first group, which we call the Anomalous Cities, consists of Arizona, Nevada, Florida, and inland California; these cities comprise a subset of the "sand state" cities in Davidoff (2013). The other large price increases occurred in the Inelastic Cities of Boston, Providence, New York, Philadelphia, and the west coast of the United States; these cities comprise a subset of the "coastal" cities in Davidoff (2013).

The history of construction and house prices in the Anomalous Cities before 2000 constitutes a puzzle for models in which a city's housing supply elasticity is constant. As shown in Figure 5, Panels B and C, from 1980 to 2000, these cities provided clear examples of very elastic housing markets in which prices stay low through rapid construction activity. Construction far outpaced the U.S. average, while house prices remained constant. In a model like Glaeser, Gyourko, and Saiz (2008), in which each city is characterized by constant housing supply elasticity, the subsequent surge in house prices in these cities is impossible. Perfectly elastic supply should have held down house prices by meeting whatever housing demand shock arrived in 2000 with higher construction.

[28] Gao, Sockin, and Xiong (2015) show that price growth during the boom displays a nonmonotonic relationship with respect to Saiz's (2010) measures of long-run supply elasticity.

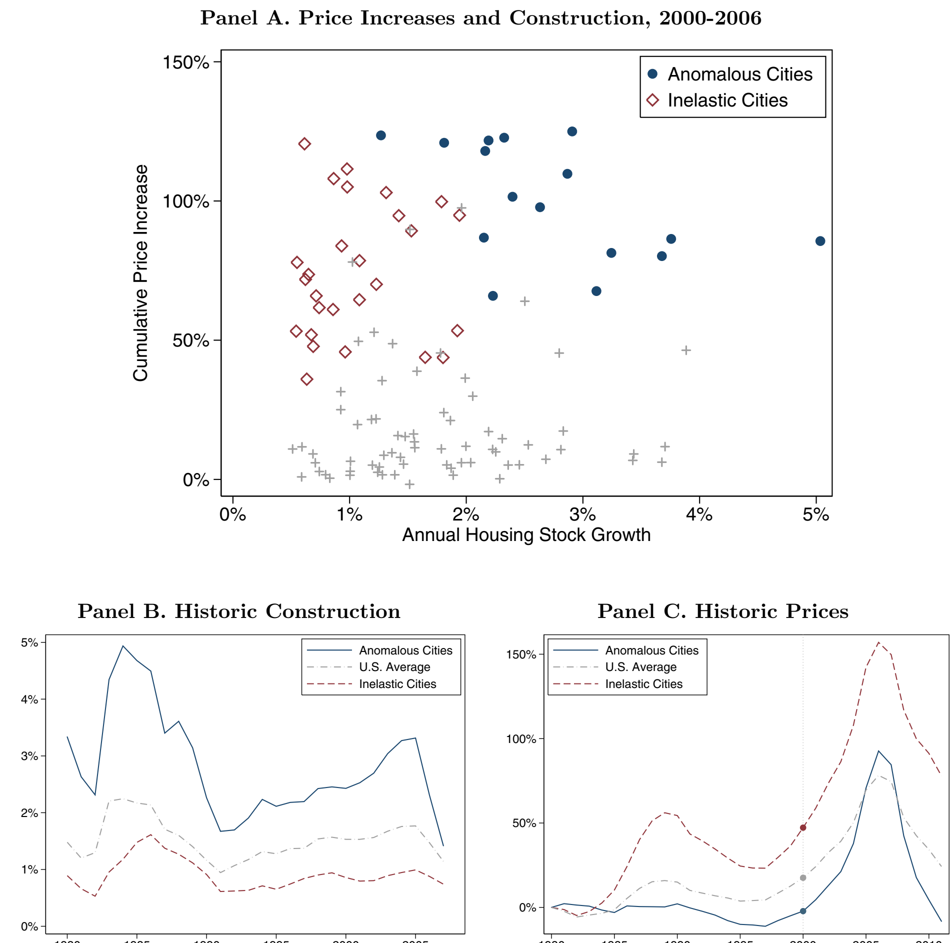

Figure 5. The U.S. housing boom and bust across cities. Anomalous Cities include those in Arizona, Nevada, Florida, and inland California. Inelastic Cities are Boston, Providence, New York, Philadelphia, and all cities on the west coast of the United States We measure the housing stock in each city at an annual frequency by interpolating the U.S. Census's decadal housing stock estimates with its annual housing permit figures. House price data come from the second-quarter FHFA house price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data. Panel A: The cumulative price increase is the ratio of the house price in 2006 to the house price in 2000. Annual housing stock growth is given as the log difference in the housing stock in 2006 and 2000 divided by six. Panels B and C: Each series is an average over cities in a group weighted by the city's housing stock in 2000. Construction is annual permitting as a fraction of the housing stock. Prices represent cumulative returns as of 1980 on the housing in each group. (Color figure can be viewed at wileyonlinelibrary.com)

Our model explains this pattern by distinguishing short-run and long-run housing supply elasticities. Proposition 1 shows that prices rise in intermediate cities in which vacant land remains because supply is expected to become constrained soon. This phenomenon depends on the way we model housing supply and holds even without disagreement. Figure 1 demonstrates long-run barriers in Las Vegas. More broadly, the land price increases across the country shown in Figure 3 indicate the presence of these constraints in other cities, or at least developers' anticipation of them.

The price increases in the Anomalous Cities were as large as those in the Inelastic Cities. The Inelastic Cities consist of markets where house prices rise because regulation and geography prohibit construction from absorbing higher demand. We document this relationship in Figure 5, Panels B and C, which show that construction in these cities was lower than the U.S. average before 2000, while house price growth greatly exceeded the U.S. average. As shown in Figure 5, Panel A, house prices increased as much in the Anomalous Cities as they did in the Inelastic Cities. Under the assumption of a common demand shock, this pattern poses a puzzle without disagreement: Proposition 1 shows that, without disagreement, constrained cities experience larger house price booms than intermediate and unconstrained cities.

Proposition 5 explains the pattern. With disagreement $z > 0$, the same demand shock $x > 0$ raises the $t = 0$ house price most in intermediate cities, not in constrained cities. According to our model, land availability in the Anomalous Cities facilitated speculation and thus amplified the increase in house prices. This amplification effect was smaller in the Inelastic Cities, which featured less undeveloped land. Evidence of disagreement during the boom comes from the stylized facts about public home builders in Section B, as Proposition 7 shows that these facts are guaranteed to hold only with disagreement.

A third puzzle is that some elastic cities built housing quickly during the boom but, unlike the Anomalous Cities, experienced stable house prices. These cities appear in the bottom-right corner of Figure 5, Panel A, and are located mostly in the southeastern United States (e.g., Texas and North Carolina).[29] Their construction during the boom quantitatively matches that in the Anomalous Cities, but the price changes are significantly smaller. Why was rapid construction able to hold down house prices in some cities and not others?

Propositions 1 and 5 explain that what distinguishes these cities are their long-run supply elasticities. A city can have perfectly elastic short-run supply, yet its long-run supply can be indeterminate. Among the cities with elastic short-run supply, the intermediate cities face constraints soon, while the unconstrained cities do not. The model's explanation of Figure 5, Panel A, is that the Anomalous Cities are the ones approaching long-run constraints,

[29] The cities with annual housing stock growth above 2% and cumulative price increases below 25% are Atlanta, Austin, Charlotte, Colorado Springs, Columbus, Dallas, Denver, Des Moines, Fort Collins, Fort Worth, Houston, Indianapolis, Lexington, Nashville, Ogden, Raleigh, and San Antonio.

whereas the cities in the bottom right did not face development barriers in the foreseeable future.

Some evidence consistent with this argument comes from the financial statements of Pulte, one of the home builders studied in Section B. In a February 2004 presentation to investors, Pulte listed several of the Anomalous Cities as "supply constrained markets you may not have expected": West Palm Beach, Orlando, Tampa, Ft. Myers, Sarasota, and Las Vegas (Chicago was also listed). In contrast, Pulte stated that Texas was "the only area of the country without supply constraints in some form," and listed many of the nonanomalous elastic cities (Atlanta, Charlotte, and Denver) as "not supply constrained overall," although "supply issues in preferred submarkets" were noted.[30] The slides appear in Section IV of the Internet Appendix.[31]

An alternate explanation for these cross-sectional patterns is that the Anomalous Cities simply experienced much larger demand growth between 2000 and 2006 than the rest of the country. Abnormally large demand growth would increase prices and construction, leading the Anomalous Cities to occupy the top-right part of Figure 5, Panel A.[32] While the application of our model considers the case of a common demand shock, Figure 5 makes it clear that demand growth did differ across cities. The cluster of cities in the bottom left of the graph likely saw low price growth and construction because demand was flat during this time. The experiences of these cities raise the possibility that the Anomalous Cities saw abnormally large demand growth just as these cities saw abnormally small growth.[33]

We examine whether any abnormal demand shocks experienced by the Anomalous Cities are sufficient to account for the extreme price movements in these cities. Mian and Sufi (2009) argue that the shock was the expansion of credit to low-income borrowers. It is possible that this shock affected the Anomalous Cities more than the rest of the nation because, for instance, they contained greater shares of low-income individuals, and that this greater exposure to the shock led to abnormally large price increases.

[30] Other cities with supply constraints only in submarkets were Phoenix, Jacksonville, Detroit, and Minneapolis.

[31] The Pulte slides provide narrative support for some of the other assumptions and predictions of the model. Pulte stated that "[Anti-growth efforts] are not new for heavily populated areas (Northeast, California) but now are widespread across the country." This statement indicates that at least one major developer—the largest public home builder at the time—recognized the rise of supply restrictions throughout the country, consistent with our assumption of finite long-run land supply.

[32] Another explanation is that the value of the option, described by Titman (1983) and Grenadier (1996), to develop land with different types of housing may have been largest in the anomalous cities, but many of these areas consist of homogeneous sprawl (Glaeser and Kahn (2004)), lessening this concern.

[33] Section D was silent on the model's predictions for construction, which we present in Section IV of the Internet Appendix. The model is ill-suited to explain construction between 2000 and 2006 because it considers a shock to news about future demand but does not consider a shock to news about current demand. As shown in Section IV of the Internet Appendix, the shock x does not alter the equilibrium housing stock, except in intermediate cities with disagreement where $x > 0$ lowers the stock relative to the case without disagreement.

To address this possibility, we calculate the house price booms that would be predicted from each city's supply elasticity and relevant demographics in 2000. We construct the predicted price increases in the following manner. Suppose that, between 2000 and 2006, each city experienced a permanent increase in log housing demand equal to x_j. From Proposition 9, the resulting increase in house prices when $z = 0$ equals

$$\Delta \log p_j^h = \frac{\overline{\mu} x_j}{2\left(\epsilon_j^s + \epsilon\right)} \tag{32}$$

to the first order in x, where ϵ_j^s is city j's housing supply elasticity. Because we are exploring a counterfactual without disagreement, this specification assumes that $\overline{\mu}$ does not vary across cities.

Mian and Sufi (2009) show that the following demographic variables predict the presence of subprime borrowers at the ZIP code level: household income (negatively), poverty rate, fraction with less than high school education, and fraction nonwhite. We measure these variables at the metropolitan area level in the 2000 U.S. Census, and use them to predict the unobserved shock x_j. We denote this vector of demographics, plus a constant and log population, by $\mathbf{d}_j$. Under the null hypothesis that these demographics alone predict the shock, we can write $\overline{\mu} x_j/2 = \beta \mathbf{d}_j + \eta_j$, where β is the same across cities, and $\mathbf{d}_j \perp \eta_j$. Substituting this expression into equation (32) yields the estimating equation

$$\left(\epsilon_j^s + \epsilon\right) \Delta \log p_j^h = \beta \mathbf{d}_j + \eta_j. \tag{33}$$

Estimating β using this equation allows us to calculate the house price boom predicted by the supply elasticity ϵ_j^s and the demographics $\mathbf{d}_j$. In equation (33), the left-hand side represents the house price increase adjusted by the elasticity of supply, while $\beta \mathbf{d}_j$ is the housing demand shock predicted by the city's exposure to subprime. We use Saiz's (2010) supply elasticity estimates for ϵ_j^s, and a value of 0.6 for the housing demand elasticity ϵ; this value lies in the range of estimates calculated by Hanushek and Quigley (1980). Using these data, we produce an estimate $\widehat{\beta}$ using ordinary least squares in equation (33). The resulting house price boom predicted from demographics and supply elasticity equals

$$\mathbf{E}\left(\Delta \log p_j^h \mid \mathbf{d}_j, \epsilon_j^s\right) = \frac{\widehat{\beta} \mathbf{d}_j}{\epsilon_j^s + \epsilon}. \tag{34}$$

Figure 6 plots the actual house price growth against the predicted price growth for each city in Figure 5, Panel A. The Anomalous Cities remain clear outliers.[34] Abnormal demand growth from low-income borrowers does not explain the extreme experiences of these cities. In theory, these predicted price

[34] Table IA.I in the Internet Appendix presents regression coefficients corresponding to Figure 6. The table shows that the degree to which the Anomalous Cities are outliers is only slightly smaller when demographics are controlled.

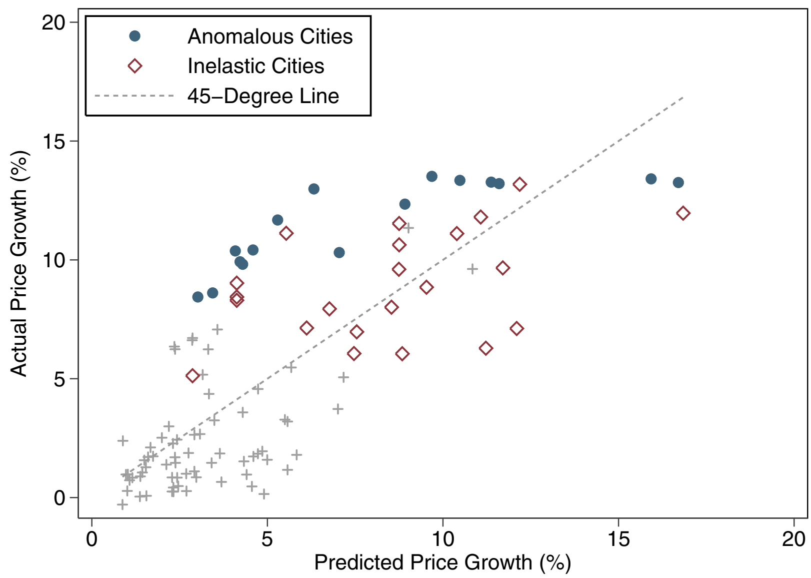

Figure 6. Anomalous cities and differential demand shocks, 2000–2006. This figure compares actual price growth during the boom to predicted price growth as a function of city-level demographics, where predicted price growth proxies for differential demand shocks. Actual price growth is the annualized log change in the second-quarter FHFA house price index deflated by the CPI-U. We compute predicted price growth from a cross-sectional regression of actual price growth on a set of city-level demographics: log population, log of median household income, percent white, percent white and not Hispanic, percent with less than 9th grade education, percent with less than 12th grade education, percent unemployed, and percent of families under the poverty line. Demographics come from the 2000 Census. (Color figure can be viewed at wileyonlinelibrary.com)

increases could have lined up well with the actual increases in the Anomalous Cities. This alignment would have held if the subprime demographics predicted the shocks, these cities were very exposed to subprime, and their housing supply elasticities were low enough. This story fails to explain the anomalous house price booms, which experienced higher price growth despite elastic supply and even conditioning on observable drivers of demand. Furthermore, the growth in subprime credit was widespread, with high-housing-supply elasticity cities experiencing large expansions in subprime credit without house price growth (Mian and Sufi (2009), Table VII).

D. Variation in House Price Booms within Cities

Proposition 8 of the model predicts larger price increases in market segments within a city that attract more renters than owners. A sufficient statistic for this effect is χ, the share of the housing stock that is rented. Proposition 8 holds only among segments with the same N_0, x, and z. This "all else equal" assumption is unlikely to hold empirically, so our discussion focuses on the conceptual predictions about within-city variation made by the model.

We first consider variation in χ across neighborhoods. Neighborhoods provide an example of market segments because they differ in the amenities they offer. For instance, some areas offer proximity to restaurants and nightlife, while others provide access to good public schools. These amenities appeal to different groups of residents. Variation in amenities therefore leads χ to vary across space. Neighborhoods whose amenities appeal relatively more to renters than to owner-occupants are characterized by a higher value of χ.

We obtain ZIP-level data on χ from the U.S. Census, which reports the share of occupied housing that is rented, as opposed to owner-occupied, in each ZIP code in 2000. The value of χ varies considerably within cities. Its national mean is 0.29 and standard deviation is 0.17, while the R^2 of regressing χ on city fixed effects is only 0.12. We calculate the real increase in house prices from 2000 to 2006 using Zillow.com's ZIP-level house price indices. We regress this price increase on χ and city fixed effects, and find a positive and highly significant coefficient of 0.10 (0.026), where the standard error is clustered at the city level. Thus, consistent with Proposition 8, house prices increased more between 2000 and 2006 in neighborhoods where χ was higher in 2000.

This positive relationship between χ and price increases may not be causal. Housing demand shocks in the boom were larger in neighborhoods with a higher value of χ. The housing boom resulted from an expansion of credit to low-income households (Mian and Sufi (2009), Landvoigt, Piazzesi, and Schneider (2015)). As a result, the strong covariance of ZIP-level income with χ will tend to bias our estimates.[35] Furthermore, a city-wide demand shock might raise house prices most strongly in cheap areas due to gentrification dynamics (Guerrieri, Hartley, and Hurst (2013)), and χ covaries negatively with the level of house prices within a city.

The appeal of χ is that it predicts price increases in any housing boom in which there is disagreement about future fundamentals. In general, χ predicts price increases because it is positively correlated with speculation, not because it is correlated with demand shocks. Empirical work can test Proposition 8 by examining housing booms in which the shocks are independent of χ.

The second approach to measuring χ is to exploit variation across different types of housing structures. According to the U.S. Census, 87% of occupied detached single-family houses in 2000 were owner-occupied rather than rented. In contrast, only 14% of occupied multifamily housing was owner-occupied. According to Proposition 8, the large difference in χ between these two types of housing causes a larger price boom in multifamily housing, all else equal.

This result squares with accounts of heightened investment activity in multifamily housing during the boom.[36] For instance, a consortium of investors—including the Church of England and California's pension fund

[35] The IRS reports the median adjusted gross income at the ZIP level. We take out city-level means, and the resulting correlation with χ is -0.40.

[36] Bayer et al. (2015) develop a method to identify speculators in the data. A relevant extension of their work would be to look at the types of housing speculators invest in.

CalPERS—purchased Stuyvesant Town & Peter Cooper Village, Manhattan's largest apartment complex, for a record price of $5.4 billion in 2006. Their investment went into foreclosure in 2010 as the price of this complex sharply fell (Segel et al. (2011)). Multifamily housing attracts speculators because it is easier to rent out than single-family housing. During periods of uncertainty, optimistic speculators bid up multifamily house prices and cause large price booms in this submarket.

VI. Conclusion

In this paper, we argue that disagreement explains an important part of housing cycles. Disagreement amplifies house price booms by biasing prices toward optimistic valuations. Our emphasis on how disagreement interacts with long-run development constraints allows us to explain aspects of the boom that are at odds with existing theories of house prices. Many of the largest price increases occurred in cities that were able to build new houses quickly. This fact poses a problem for theories that stress inelastic housing supply as the sole source of house price booms, but it sits well with our theory, which instead emphasizes speculation. Undeveloped land facilitates speculation due to rental frictions in the housing market. In our model, large price booms occur in elastic cities facing a development barrier in the near future.

Introducing key aspects of the housing market—heterogeneous ownership utility and the nature of asset supply—extends and clarifies past work in finance that has focused on disagreement in financial asset markets. In our model, disagreement raises the price of housing only under certain conditions and the relationship between disagreement and asset supply can be nonmonotonic. The particular setting of housing markets also presents an important case in which disagreement reduces welfare, as pessimists with high flow utility may be replaced by optimists with low flow utility.

We document the central importance of land price increases for explaining the U.S. house price boom between 2000 and 2006. These land price increases resulted from speculation directly in the land market. Consistent with this theory, home builders significantly increased their land investments during the boom and then suffered large capital losses during the bust. Many investors disagreed with this optimistic behavior and short-sold home builder equity as the homebuilders were buying land.

In one of the model's extensions, price booms are larger in submarkets within a city where a greater share of housing is rented. We present some evidence for this prediction, but further empirical work is needed to test it more carefully. We also look forward to work exploring these findings to understand cycles outside the United States, in historical episodes, and in other markets with similar features to housing.

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Appendix A

PROOF OF LEMMA 1: If $p_1^h > p_1^l + k$, then each developer wants to buy an infinite amount of land, build houses with the land, and then sell the houses. As a result, the land market cannot clear. If $p_1^h < p_1^l + k$, then the reverse holds, that is, developers want to sell an infinite amount of land. The land market cannot clear in this case either. In equilibrium, the only possibility is that $p_1^h = p_1^l + k$. At $t = 1$, demand from arriving potential residents equals $N_1 SD(p_1^h)$. Supply from outgoing residents equals zero if $p_1^h < 0$ and Q_r if $p_1^h > 0$, where Q_r is the number of potential residents who bought at $t = 0$. Developers are indifferent to how much housing they sell because $p_1^h = p_1^l + k$, but the most they can sell emerges from summing across the two developer constraints to obtain $\sum H_1^{\text{sell}} \leq \sum H_1 + L_1 = S - Q_r$. The sum of H_1^{sell} across developers and potential residents cannot exceed S.

We now consider three possible equilibria. In the first, $p_1^l < 0$. This inequality cannot hold in equilibrium because developer land demand would be infinite for each developer, and the land market would not clear. The next possibility is that $p_1^l = 0$. In this case, demand from arriving potential residents equals $N_1 S$. If $N_1 > 1$, then this equilibrium fails because maximal aggregate home sales equal S. If $N_1 \leq 1$, then we construct an equilibrium as follows. We cannot have $N_1 S > L_1 + H_1 + Q_r$ for all developers (summing across them delivers a contradiction when $N_1 \leq 1$), so consider a developer for whom $N_1 S \leq L_1 + H_1 + Q_r$. This developer sets $L_1^{\text{buy}} = 0$, $H_1^{\text{build}} = N_1 S - Q_r - H_1$, and $H_1^{\text{sell}} = N_1 S - Q_r$. All other developers set $L_1^{\text{buy}} = 0$, $H_1^{\text{build}} = -H_1$, and $H_1^{\text{sell}} = 0$. All developer constraints and optimality conditions are satisfied under these choices, and both the housing and land markets clear. Finally, we consider the possibility that $p_1^l > 0$. Because $p_1^h > 0$, the first constraint for the developers binds, so we can rewrite the developer objective function as $p_1^h H_1 + p_1^l (H_1^{\text{build}} - L_1^{\text{buy}})$. Because $p_1^l > 0$, the developer maximizes this objective by satisfying the second constraint and setting $H_1^{\text{build}} - L_1^{\text{buy}} = L_1$. Because both constraints are satisfied with equality, summing across them yields $\sum H_1^{\text{sell}} = \sum H_1 + \sum L_1 + \sum L_1^{\text{buy}}$. If the land market clears, the last sum equals zero. The housing market clears when $N_1 SD(p_1^h) = \sum H_1 + \sum L_1 + Q_r = S$. If $N_1 < 1$, then no solution for p_1^h exists. If $N_1 = 1$, then the only solutions have $p_1^h \leq k$, contradicting the assumption that $p_1^l > 0$. This equilibrium is possible if and only if $N_1 > 1$, in which case the unique solution is $p_1^h = kN_1^{1/\epsilon}$. The optimality conditions and constraints for all developers are satisfied if they set $H_1^{\text{build}} = L_1$, $H_1^{\text{sell}} = H_1 + L_1$, and $L_1^{\text{buy}} = 0$. The land and housing markets clear under these choices. In summary, a unique equilibrium exists for each value of N_1. When $N_1 \leq 1$, only $p_1^l = 0$ and $p_1^h = k$ are possible, whereas when $N_1 > 1$, only $p_1^l = kN_1^{1/\epsilon} - k$ and $p_1^h = N_1^{1/\epsilon}$ are possible.

PROOF OF LEMMA 2: The utility at $t = 1$ of a resident who bought at $t = 0$ equals $p_1^h - p_0^h + v$ if $p_1^h \geq 0$. Housing demand from potential residents at $t = 0$ equals $\int_\Theta N_0 SD(p_0^h - p_1^h(e^{\mu(\theta)x} N_0)) f_r(\theta) d\theta$. For the same argument given in the proof of Lemma 1 that $p_1^h = p_1^l + k$, $p_0^h = p_0^l + 2k$ in equilibrium; developers would want to buy or sell infinite land otherwise. In all of the equilibria characterized in the proof of Lemma 1, $\pi = p_1^h H_1 + p_1^l L_1 + B_1$. By making substitutions using the constraints of the $t = 0$ developer problem, we see that the objective at $t = 0$ is to choose $H_1, L_1 \geq 0$ to maximize $(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h)H_1 + (p_1^h(e^{\mu(\theta)x}N_0) - p_0^h + k)L_1 + p_0^l L_0$. In all equilibria, all developers choose finite values of H_1 and L_1, so the first-order conditions imply $p_1^h(e^{\mu(\theta)x}N_0) - p_0^h + k \leq 0$ for all $\theta \in \operatorname{supp} f_d$. Because $p_1^h(\cdot)$ increases, either $p_0^h = p_1^h(e^{\mu_d^{\max}x}N_0) + k$, in which case developers with $\theta = \theta_d^{\max}$ may choose any $L_1 \geq 0$, or $p_0^h > p_1^h(e^{\mu_d^{\max}x}N_0) + k$, in which case $L_1 = 0$ for all developers.

We now consider these two possible equilibria. The first may hold only if potential resident housing demand does not exceed S (developers cannot build more than this quantity of housing, and the housing market must clear). This condition reduces to

$$1 \geq N_0 \int_\Theta D\left(p_1^h(e^{\mu_d^{\max}x}N_0) + k - p_1^h(e^{\mu(\theta)x}N_0)\right) f_r(\theta)d\theta. \tag{A1}$$

If (A1) holds, then we construct an equilibrium as follows. Let $Q_r = \int_\Theta N_0 S D(p_1^h(e^{\mu_d^{\max}x}N_0) + k - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta)d\theta$. For one developer for whom $\theta = \theta_d^{\max}$, we set $H_0^{\text{build}} = Q_r$, $H_0^{\text{sell}} = Q_r$, and $L_0^{\text{buy}} = S - L_0$. For all other developers, we set $H_0^{\text{build}} = 0$, $H_0^{\text{sell}} = 0$, and $L_0^{\text{buy}} = -L_0$. All developer constraints and optimality conditions hold, and the housing and land markets clear.

If (A1) fails, then we must have $N_0 > 1$ because $D \leq 1$. As a result, we may define $p_0^h(N_0, x, z)$ to be the unique solution to

$$1 = N_0 \int_\Theta D\left(p_0^h - p_1^h(e^{\mu(\theta)x}N_0)\right) f_r(\theta)d\theta. \tag{A2}$$

To see that a solution to (A2) exists, consider that when $p_0^h = 2k$, the right side of the equation equals $N_0 > 1$. Because the right side goes to zero (pointwise) as $p_0^h \to \infty$, by the intermediate value theorem, we may find p_0^h satisfying (A2). This solution is unique because the integrand strictly decreases if $p_0^h - p_1^h(e^{\mu(\theta)x}N_0) \geq k$; this equation must hold at $p_0^h = p_0^h(N_0, x, z)$ for at least some $\theta \in \operatorname{supp} f_r$ for otherwise the right side of (A2) exceeds one. Because the right side of (A2) weakly decreases in p_0^h, if (A1) fails, then $p_0^h(N_0, x, z) > p_1^h(e^{\mu_d^{\max}x}N_0) + k$. As a result, developer constraints and optimality conditions are satisfied when for all developers $L_0^{\text{buy}} = 0$, $H_0^{\text{build}} = L_0$, and $H_0^{\text{sell}} = L_0$; housing and land markets clear as well.

In summary, if (A1) holds, then the unique equilibrium price is $p_0^h(N_0, x, z) = p_1^h(e^{\mu_d^{\max}x}N_0) + k$; if (A1) fails, then the unique equilibrium price $p_0^h(N_0, x, z)$ is the unique solution to (A2).

PROOF OF PROPOSITION 1: From the proof of Lemma 2, developers hold land at the end of $t = 0$ if and only if (A1) holds without equality. When $z = 0$, $\mu(\theta) = \mu_d^{\max}$ for all $\theta \in \Theta$, so (A1) reduces to $1 \geq N_0$. As a result, developers hold land at the end of $t = 0$ if and only if $N_0 < 1$, as claimed. In this case, $p_0^h(N_0, x, 0) = p_1^h(e^{\bar{\mu}x}N_0) + k$. If $N_0 < e^{-\bar{\mu}x}$, then (from Lemma 1)

$p_0^h(N_0, x, 0) = 2k$; if $e^{-\overline{\mu}x} \leq N_0 < 1$, then $p_0^h(N_0, x, 0) = k(e^{\overline{\mu}x/\epsilon} N_0^{1/\epsilon} - 1)$. If $N_0 \geq 1$, then $p_0^h(N_0, x, 0) = kN_0^{1/\epsilon} + p_1(e^{\overline{\mu}x} N_0)$ is the unique solution to (A2). Because $\overline{\mu}x \geq 0$, in this case $p_0^h(N_0, x, 0) = k(1 + e^{\overline{\mu}x/\epsilon})N_0^{1/\epsilon}$, as claimed. The final equation in the proposition follows from the solution for $p_0^h(N_0, x, 0)$; note that, in the intermediate case, if $x = 0$, then $p_0^h = 2k$ as in the unconstrained case.

Proof of Proposition 2: The number of potential residents of type θ who purchase housing equals $N_0 SD(p_0^h(N_0, x, z) - p_1(e^{\mu(\theta)x} N_0)) > 0$, so Assumptions 1 and 2 guarantee that residents of each type $\theta \in \operatorname{supp} f_r$ hold housing. To prove the other parts of the proposition, we show that there exists a unique $N_0^*(x, z) \in \mathbb{R}_{>1} \cup \{\infty\}$ such that (A1) holds strictly if and only if $N_0 < N_0^*(x, z)$ and with equality if and only if $N_0 = N_0^*(x, z)$. We may rewrite (A1) as

$$1 \geq \begin{cases} N_0 & \text{if } N_0 \leq e^{-\mu_d^{\max} x} \\ \int_{\Theta_1(N_0,x,z)} N_0 f_r(\theta) d\theta + \int_{\Theta_2(N_0,x,z)} e^{-\mu_d^{\max} x} f_r(\theta) d\theta & \\ + \int_{\Theta_3(N_0,x,z)} \left(N_0^{-1/\epsilon} + e^{\mu_d^{\max} x/\epsilon} - e^{\mu(\theta)x/\epsilon} \right)^{-\epsilon} f_r(\theta) d\theta & \text{if } N_0 > e^{-\mu_d^{\max} x}, \end{cases} \tag{A3}$$

where $\Theta_1(N_0, x, z) = \{\theta \mid \theta \geq \theta_d^{\max}\}$, $\Theta_2(N_0, x, z) = \{\theta \mid \theta < \theta_d^{\max}\} \cap \{\theta \mid e^{\mu(\theta)x} N_0 \leq 1\}$, and $\Theta_3(N_0, x, z) = \{\theta \mid \theta < \theta_d^{\max}\} \cap \{\theta \mid e^{\mu(\theta)x} N_0 > 1\}$. For notational ease, we name the right side of this inequality $\phi(N_0)$. We have $\lim_{N_0 \to 0} \phi(N_0) = 0$, and ϕ strictly increases for $0 < N_0 \leq e^{-\mu_d^{\max} x}$. The integrands coincide for θ in the boundary of Θ_2 and Θ_3, so $\phi'(N_0)$ for $N_0 \geq e^{-\mu_d^{\max} x}$ equals the sum of the derivatives under the integral signs (the changing limits of integration cancel out). Therefore, ϕ strictly increases in N_0 for all $N_0 > 0$ except those for which $\Theta_2(N_0, x, z) = \operatorname{supp} f_r$. For any such N_0, $1 > \phi(N_0)$ because $\mu_d^{\max} > \overline{\mu} \geq 0$ given Assumption 4 and given that $z > 0$. The increasing nature of ϕ means that there may exist at most one solution to $1 = \phi(N_0)$, and that (A1) is satisfied strictly for any N_0 less than this solution and is not satisfied for any N_0 greater than this solution. We call the solution $N_0^*(x, z)$. Note that $\phi(1) < 1$ unless $\int_{\Theta_1(1,x,z)} f_r(\theta) d\theta = 1$, which is impossible by Assumption 4. Therefore, $\phi(1) < 1$ and $N_0^*(x, z) > 1$. For later proofs, we note here that $\lim_{z \to 0} N_0^*(x, z) = 1$, which is evident because, for any $N_0 > 1$, $\lim_{z \to 0} \phi(N_0) > 1$, while $\phi(1) < 1$ for all $z > 0$.

The existence of $N_0^*(x, z)$ implies that some developers hold land if and only if $N_0 < N_0^*(x, z)$. If $N_0 < N_0^*(x, z)$, then the proof of Lemma 2 shows that $p_0^h = p_1^h(e^{\mu_d^{\max} x} N_0) + k$ and that a developer of type θ may hold land if and only if $p_1^h(e^{\mu(\theta)x} N_0) + k \leq p_0^h(N_0, x, z)$. If $N_0 \leq e^{-\mu_d^{\max} x}$, then $p_1^h(e^{\mu(\theta)x} N_0) = k$ for all $\theta \in \operatorname{supp} f_d$, so any developer may hold land at the end of $t = 0$, as claimed. If $e^{-\mu_d^{\max} x} < N_0 < N_0^*(x, z)$, then the only developers for whom $p_1^h(e^{\mu(\theta)x} N_0) \leq p_1^h(e^{\mu_d^{\max} x} N_0)$ are those for whom $\theta = \theta_d^{\max}$ due to Lemma 1. As a result, only these developers hold land when N_0 satisfies these constraints, as claimed.

We now prove the result on excess land holdings by developers in the intermediate case. Define Q_r to be the quantity of housing held by potential residents in equilibrium. From Lemma 2, Q_r does not depend on the developer land endowments L_0, and by the first part of the proposition, $0 < Q_r < S$ for $e^{-\mu_d^{\max} x} < N_0 < N_0^*(x, z)$. Summing across the constraint on L_1 for developers at $t = 0$ with $\theta = \theta_d^{\max}$ yields $S - Q_r = \sum L_1 = \sum(L_0 + (L_0^{\text{buy}})^* - (H_0^{\text{build}})^*)$. As a result, $\sum((L_0^{\text{buy}})^* - (H_0^{\text{build}})^*) = S - Q_r - \sum L_0$, which exceeds zero as long as $\sum L_0 < S - Q_r \equiv L^*$.

Finally, if $\int_{\theta \geq \theta_d^{\max}} f_r(\theta)d\theta > 0$, then $\lim_{N_0 \to \infty} \phi(N_0) = \infty$, leading to $N_0^*(x, z) < \infty$. If $\int_{\theta \geq \theta_d^{\max}} f_r(\theta)d\theta = 0$, then $\lim_{N_0 \to \infty} \phi(N_0) = \int_{\theta < \theta_d^{\max}} (e^{\mu_d^{\max} x/\epsilon} - e^{\mu(\theta)x/\epsilon}) f_r(\theta)d\theta$, so $N_0^*(x, z) < \infty$ in this case if and only if this integral exceeds one.

PROOF OF LEMMA 3: First, we prove existence and uniqueness of $\mu_r^{\text{agg}}(N_0, x, z)$. For $N_0 \geq N_0^*(x, z)$, $p_0^h(N_0, x, z) \geq p_1^h(e^{\mu_d^{\max} x} N_0) + k$ as shown in the proof of Lemma 2. Because $\mu_d^{\max} > \overline{\mu} \geq 0$, $p_1^h(e^{\mu_d^{\max} x} N_0) = k e^{\mu_d^{\max} x/\epsilon} N_0^{1/\epsilon}$, so $p_0^h(N_0, x, z) > k N_0^{1/\epsilon}$. It follows that $p_0^h(N_0, x, z) = k(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon}) N_0^{1/\epsilon}$ has a unique solution for $\mu_r^{\text{agg}}(N_0, x, z)$, as the right side strictly increases from $k N_0^{1/\epsilon}$ to ∞ as $\mu_r^{\text{agg}}(N_0, x, z)$ approaches ∞ (which holds due to the assumption that $x > 0$).

We next show that $\mu_r^{\text{agg}}(N_0, x, z) < \mu_r^{\max}$. Because the right side of (25) weakly decreases in μ_r^{agg}, it suffices to show that

$$1 > N_0 \int_{\Theta} D\left(k\left(1 + e^{\mu_r^{\max} x/\epsilon}\right) N_0^{1/\epsilon} - p_1^h(e^{\mu(\theta)x} N_0)\right) f_r(\theta)d\theta. \tag{A4}$$

Due to Assumption 4, $\mu_r^{\max} > 0$ and $\int_{\theta < \theta_r^{\max}} f_r(\theta)d\theta > 0$. Because $p_1^h(\cdot)$ strictly increases on $[1, \infty)$, $p_1^h(e^{\mu(\theta)x} N_0) < p_1^h(e^{\mu_r^{\max} x} N_0)$ for $\theta < \theta_r^{\max}$. When $\theta = \theta_r^{\max}$, the argument of $D(\cdot)$ in (A4) equals $k N_0^{1/\epsilon}$. Because $D(\cdot)$ strictly decreases on $[k, \infty)$, it follows that the right side of (A4) is strictly less than $N_0 D(k N_0^{1/\epsilon}) = 1$, as desired.

We next use the conditions in the lemma to simplify (25). Because $p_0^h(N_0, x, z) \geq p_1^h(e^{\mu_d^{\max} x} N_0) + k$ for $N_0 \geq N_0^*(x, z)$ (as shown in the proof of Lemma 2), for all $\theta \in \operatorname{supp} f_r$ we may bound the argument of $D(\cdot)$ in (25) as follows: $k(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon}) N_0^{1/\epsilon} - p_1^h(e^{\mu(\theta)x} N_0) \geq k + p_1^h(e^{\mu_d^{\max} x} N_0) - p_1^h(e^{\mu(\theta)x} N_0) \geq k$, with the last inequality holding because $\theta \leq \theta_d^{\max}$ by assumption. Furthermore, because $\theta \geq -\overline{\mu}/z$ for all $\theta \in \operatorname{supp} f_r$, $\mu(\theta) \geq 0$ for all such θ, allowing us to write $p_1^h(e^{\mu(\theta)x} N_0) = k e^{\mu(\theta)x/\epsilon} N_0^{1/\epsilon}$ for all $\theta \in \operatorname{supp} f_r$. It follows from Assumption 1 that $\mu_r^{\text{agg}}(N_0, x, z)$ must solve

$$1 = \int_{\operatorname{supp} f_r} (1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon} f_r(\theta)d\theta. \tag{A5}$$

We finally show that $\mu_r^{\text{agg}}(N_0, x, z) > \overline{\mu}$ and that $\mu_r^{\text{agg}}(N_0, x, z) = \overline{\mu} + o(z)$ as $z \to 0$. The argument of the integral in (A5) is convex in θ for $\theta \in \operatorname{supp} f_r$, so Jensen's inequality implies that $1 > (1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} - e^{\overline{\mu}x/\epsilon})^{-\epsilon}$, from which it follows that $\mu_r^{\text{agg}}(N_0, x, z) > \overline{\mu}$. Taking the derivative of (A5) with respect to z and simplifying yields

$$\frac{\partial \mu_r^{\text{agg}}(N_0, x, z)}{\partial z} = \frac{\int_{\operatorname{supp} f_r} \theta e^{\mu(\theta)x/\epsilon}(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon-1} f_r(\theta)d\theta}{\int_{\operatorname{supp} f_r} e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} \left(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon-1} f_r(\theta)d\theta}. \tag{A6}$$

As $z \to 0$, the denominator goes to $e^{\overline{\mu}x/\epsilon}$, whereas the numerator goes to $\int_\Theta e^{\overline{\mu}x/\epsilon}\theta f_r(\theta)d\theta$, which equals zero because $\int_\Theta \theta f_r(\theta)d\theta = 0$ by assumption. Therefore, $\mu_r^{\text{agg}}(N_0, x, z) = \overline{\mu} + o(z)$ as $z \to 0$.

PROOF OF PROPOSITION 3: If $N_0 < N_0^*(x, z)$, then (A1) holds and $p_0^h(N_0, x, z) = p_1^h(e^{\mu_d^{\max}x}N_0) + k$. By applying Lemma 1, we arrive at the first two pricing equations in Proposition 3. The equation for $N \geq N_0^*(x, z)$ follows immediately from (A2).

PROOF OF PROPOSITION 4: Substituting the formulas for $p_0^h(N_0, x, z)$ from Proposition 2 and for $p_0^h(N_0, x, 0)$ from Proposition 1 yields the equations in the first part of Proposition 4. For clarity, we prove the remainder of the claims in Proposition 4 in three parts.

Part 1: Disagreement effect is maximized at $N_0 = 1$: The effect of disagreement on the house price at $t = 0$ weakly increases in N_0 up to $N_0 = 1$ and decreases for $1 \leq N_0 \leq N_0^*(x, z)$. The maximum for $0 < N_0 \leq N_0^*(x, z)$ therefore equals the value at $N_0 = 1$, which is $(e^{\mu_d^{\max}x/\epsilon} - e^{\overline{\mu}x/\epsilon})/(1 + e^{\overline{\mu}x/\epsilon})$. This value exceeds the disagreement effect for all $N_0 \geq N_0^*(x, z)$ if and only if $\mu_d^{\max} > \mu_r^{\text{agg}}(N_0, x, z)$ for all $N_0 \geq N_0^*(x, z)$. We prove this in two steps. We first show that $p_0^h(N_0, x, z)$, and hence $\mu_r^{\text{agg}}(N_0, x, z)$, weakly increases in N_0 for $N_0 \geq N_0^*(x, z)$. Second, to show that $\mu_r^{\text{agg}}(N_0, x, z) < \mu_d^{\max}$ for all $N_0 \geq N_0^*(x, z)$, we show that $\lim_{N_0\to\infty} \mu_r^{\text{agg}}(N_0, x, z)$ exists and is less than $\mu_d^{\max}$.

We may rewrite (A2) as

$$
\begin{aligned}
1 = &\int_{\Theta_1(N_0,x,z)} N_0 D\left(p_0^h - k\right) f_r(\theta)d\theta \\
&+ \int_{\Theta_2(N_0,x,z)} N_0 \left(\frac{k}{p_0^h - ke^{\mu(\theta)x/\epsilon}N_0^{1/\epsilon}}\right)^{\epsilon} f_r(\theta)d\theta + \int_{\Theta_3(N_0,x,z)} N_0 f_r(\theta)d\theta,
\end{aligned} \tag{A7}
$$

where $\Theta_1(N_0, x, z) = \{\theta \mid e^{\mu(\theta)x}N_0 < 1\}$, $\Theta_2(N_0, x, z) = \{\theta \mid 1 \leq e^{\mu(\theta)x}N_0 \leq (p_0^h/k - 1)^\epsilon\}$, and $\Theta_3(N_0, x, z) = \{\theta \mid e^{\mu(\theta)x}N_0 > (p_0^h/k - 1)^\epsilon\}$. For a given p_0^h, the right side of (A7) weakly increases in N_0: each integrand weakly increases in N_0 (for each θ), and the integrands coincide at the boundaries of the limits of integration, meaning that the marginal effect from changing the limits of integration equals zero. Because the right side of (A2) weakly decreases in p_0^h (as shown in the proof of Lemma 2), it follows that $p_0^h(N_0, x, z)$ weakly increases in N_0.

This monotonicity means that for all $N_0 \geq N_0^*(x, z)$, $\mu_r^{\text{agg}}(N_0, x, z) \leq \lim_{N_0'\to\infty} \mu_r^{\text{agg}}(N_0', x, z)$. Substituting $p_0^h(N_0, x, z) = k(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon}$ into (A7) yields

$$
\begin{aligned}
1 = &\int_{\Theta_1(N_0,x,z)} \left(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} - N_0^{-1/\epsilon}\right)^{-\epsilon} f_r(\theta)d\theta \\
&+ \int_{\Theta_2(N_0,x,z)} \left(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon} f_r(\theta)d\theta + \int_{\Theta_3(N_0,x,z)} N_0 f_r(\theta)d\theta.
\end{aligned} \tag{A8}
$$

Because $\mu_r^{\text{agg}}(N_0, x, z)$ increases in N_0, either $\lim_{N_0\to\infty} \mu_r^{\text{agg}}(N_0, x, z)$ exists and is finite or it equals ∞. In the latter case, because $\mu(\theta) \le \mu_r^{\max}$ for all $\theta \in \operatorname{supp} f_r$, each integral approaches zero as $N_0 \to \infty$, leading to a contradiction. (That the last integral $\to 0$ follows because $\theta \in \Theta_3(N_0, x, z)$ if and only if $e^{\mu(\theta)x} N_0 \ge ((1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} - 1)^\epsilon$, which implies that $\mu(\theta) \ge \mu_r^{\text{agg}}(N_0, x, z)$ because $N_0 \ge N_0^*(x, z) > 1$. For all N_0 such that $\mu_r^{\text{agg}}(N_0, x, z) > \mu_r^{\max}$, $\int_{\Theta_3(N_0,x,z)} f_r(\theta) d\theta = 0$.) Thus, $\lim_{N_0\to\infty} \mu_r^{\text{agg}}(N_0, x, z) < \infty$. In this case, $\lim_{N_0\to\infty} \Theta_3(N_0, x, z) = \{\theta \mid e^{\mu(\theta)x/\epsilon} \ge 1 + e^{\lim_{N_0\to\infty} \mu_r^{\text{agg}}(N_0,x,z)x/\epsilon}\}$, whose measure under f_r must equal zero for otherwise $\lim_{N_0\to\infty} \int_{\Theta_3(N_0,x,z)} N_0 f_r(\theta) = \infty$, a contradiction due to (A8). Because $\lim_{N_0\to\infty} \Theta_1(N_0, x, z) = \emptyset$, taking the limit of (A8) as $N_0 \to \infty$ yields $1 = \int_\Theta (1 + e^{\lim_{N_0\to\infty} \mu_r^{\text{agg}}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon} f_r(\theta) d\theta$. By Assumption 5, this equation implies that $\lim_{N_0\to\infty} \mu_r^{\text{agg}}(N_0, x, z) < \mu_d^{\max}$.

Part 2: Positivity of the disagreement effect: Because $\mu_d^{\max} > \overline{\mu}$, the effect of disagreement on prices is positive for $e^{-\mu_d^{\max}x} < N_0 \le 1$. Because the effect decreases for $1 \le N_0 \le N_0^*(x, z)$, it is positive for $N_0 > 1$ if it is positive for $N_0 \ge N_0^*(x, z)$ in the case that $N_0^*(x, z) < \infty$, or if its asymptote as $N_0 \to \infty$ is positive in the case that $N_0^*(x, z) = \infty$.

In the first case, positivity obtains when $\operatorname{supp} f_r \subset [-\overline{\mu}/z, \theta_d^{\max}]$ and $\int_\Theta f_r(\theta) d\theta = 0$ because Lemma 3 shows that $\mu_r^{\text{agg}}(N_0, x, z) > \overline{\mu}$ for $N_0 \ge N_0^*(x, z)$.

In the second case, by Proposition 2 and Jensen's inequality, $1 > (e^{\mu_d^{\max}x/\epsilon} - e^{\overline{\mu}x/\epsilon})^{-\epsilon}$, which implies that $e^{\mu_d^{\max}x/\epsilon} > 1 + e^{\overline{\mu}x/\epsilon}$. When $N_0^*(x, z) = \infty$, the effect of disagreement on the house price asymptotes to $(e^{\mu_d^{\max}x/\epsilon} - e^{\overline{\mu}x/\epsilon} - 1)/(1 + e^{\overline{\mu}x/\epsilon}) > 0$.

Part 3: Marginal effect of small disagreement: For any $N_0 < e^{-\overline{\mu}x}$, we may find $z > 0$ small enough so that $N_0 < e^{-\mu_d^{\max}x}$ because $\lim_{z\to 0} \mu_d^{\max} = \overline{\mu}$. By the first part of Proposition 4, for such small z, $\partial p_0^h(N_0, x, z)/\partial z = 0$, proving the first part of the formula.

For $e^{\overline{\mu}x} \le N_0 \le 1$, we divide the formula in the first part of Proposition 4 by z and take the limit as $z \to 0$ to obtain the expression in the second part of the formula.

Finally, for each $N_0 > 1$, because $\lim_{z\to 0} N_0^*(x, z) = 1$ as shown in the proof of Proposition 2, for small enough $z > 0$, we have $N_0 \ge N_0^*(x, z)$. Because $\mu_r^{\text{agg}}(N_0, x, z) = \overline{\mu} + o(z)$ as shown by Lemma 3, $\partial \mu_r^{\text{agg}}(N_0, x, 0)/\partial z = 0$ and $\partial p_0^h(N_0, x, 0)/\partial z = 0$.

Proof of Proposition 5: As noted in the text, $p_0^h(N_0, 0, z) = p_0^h(N_0, 0, 0)$ because when $x = 0$, $\mu(\theta) = \overline{\mu}$ for all $\theta \in \Theta$, so z becomes irrelevant for the equilibrium. As a result, we may take the formula for $p_0^h(N_0, 0, 0)$ given by Proposition 1 in the special case in which $x = 0$ and use it for $p_0^h(N_0, 0, z)$. Combining this formula with that for $p_0^h(N_0, x, z)$ given by Proposition 4 yields the formulas in Proposition 5. The boom strictly increases for $e^{-\mu_d^{\max}x} \le N_0 \le 1$ and strictly decreases for $1 \le N_0 \le N_0^*(x, z)$. Therefore, it is strictly maximized at $N_0 = 1$ as long as its value at $N_0 = 1$, which equals $(e^{\mu_d^{\max}x/\epsilon} - 1)/2$, exceeds the boom for all

$N_0 \geq N_0^*(x,z)$, which occurs as long as $\mu_d^{\max} > \mu_r^{\text{agg}}(N_0,x,z)$ for all $N_0 \geq N_0^*(x,z)$. This inequality obtains by Assumption 5, as shown in the proof of Proposition 4.

PROOF OF PROPOSITION 6: As shown in the proof of Lemma 2, in equilibrium, the profit (utility for firm owners) of a developer equals $p_0^l L_0 + (p_1^h - p_0^h + k)L_1$ and the utility of potential residents equals $(v + p_1^h - p_0^h)H_0^{\text{buy}}$. The set of possible changes to the allocation is summarized by a cash transfer τ (which may vary across agents) with $\sum \tau = 0$ and changes ΔL_1 for each developer and ΔH_0^{buy} for each potential resident such that $\sum \Delta L_1 = \sum \Delta H_0^{\text{buy}}$. For a given realization of p_1^h, this change is a Pareto improvement only if $(p_1^h - p_0^h + k)\Delta L_1 + \tau \geq 0$ for all developers and $(v + p_1^h - p_0^h)\Delta H_0^{\text{buy}} + \tau \geq 0$ for all potential residents, with at least one strict inequality. Summing these inequalities across agents gives $\sum k\Delta L_1 + \sum v\Delta H_0^{\text{buy}} > 0$.

We now show that the $z = 0$ equilibrium (described in the proof of Lemma 2) is Pareto-efficient for any p_1^h. If $N_0 < 1$, a potential resident buys if and only if $v \geq p_0^h(N_0,x,z) - p_1^h(e^{\mu(\theta)x}N_0) = k$. The only feasible ΔH_0^{buy} are -1 for someone with $H_0^{\text{buy}} = 1$ and 1 for someone with $H_0^{\text{buy}} = 0$. Because $\sum \Delta L_1 = \sum \Delta H_0^{\text{buy}}$, either one of these changes does not increase the welfare criterion given above. When $N_0 \geq 1$, $L_1 = 0$ for all developers and a potential resident buys only if $v \geq p_0^h(N_0,x,z) - p_1^h(e^{\mu(\theta)x}N_0) = kN_0^{1/\epsilon} \geq k$. The only feasible ΔL_1 are positive, and the only feasible changes to ΔH_0^{buy} are 1 for $v \leq kN_0^{1/\epsilon}$ and -1 for $v \geq kN_0^{1/\epsilon}$. The change to the welfare criterion above can never be positive resulting from these changes. As a result, the allocation under the $z = 0$ equilibrium is Pareto-efficient for any p_1^h, meaning that it is belief-neutral Pareto-efficient.

When $N_0 \leq e^{-\mu_d^{\max}x}$, potential residents buy when $v \geq p_0^h(N_0,x,z) - p_1^h(e^{\mu(\theta)x}N_0)$. This difference is no greater than k because $p_0^h(N_0,x,z) = 2k$ and $p_1^h(e^{\mu(\theta)x}N_0) \geq k$. Because all potential residents have $v \geq k$ by Assumption 1, buyers all have $v \geq k$, and all potential residents with $v > k$ buy. For the same argument given in the $z = 0$ equilibrium above, there does not exist a reallocation that improves welfare for each p_1^h, meaning that the equilibrium under the equilibrium with $z > 0$ is belief-neutral Pareto-efficient when $N_0 \leq e^{-\mu_d^{\max}x}$.

When $e^{-\mu_d^{\max}x} < N_0 < N_0^*(x,z)$, $L_1 > 0$ for at least one developer. If $z > 0$, then, by Assumption 4, there exists a positive measure of potential residents for whom $\theta < \theta_d^{\max}$. These potential residents buy only if $v \geq p_0^h(N_0,x,z) - p_1^h(e^{\mu(\theta)x}N_0) > p_0^h(N_0,x,z) - p_1^h(e^{\mu_d^{\max}x}N_0) = k$, where the latter equality uses an equilibrium condition from the proof of Lemma 2. It follows that there exists a positive measure of potential residents with $v > k$ who do not buy. For a given p_1^h, we improve the allocation by setting $\Delta L_1 = -1$ and $\tau = \tau^*$ for a developer holding land and setting $\Delta H_0^{\text{buy}} = 1$ and $\tau = -\tau^*$ for a potential resident with $v > k$ who does not buy a house, where $k + p_1^h - p_0^h \leq \tau^* \leq v + p_1^h - p_0^h$.

When $N_0 \geq N_0^*(x,z)$, potential residents buy only if $v \geq k(1 + e^{\mu_r^{\text{agg}}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} - p_1^h(e^{\mu(\theta)x}N_0)$. Due to Assumption 4, $z > 0$ implies that $\mu(\theta)$ varies across potential residents. Because $N_0 > 1$ and $\overline{\mu} \geq 0$, $p_1^h(e^{\mu(\theta)x}N_0)$ varies across potential residents. It follows that the purchase cutoff varies across potential residents, meaning that we can find a potential resident with $H_0^{\text{buy}} = 1$ and $v = v^1$ and a potential resident with $H_0^{\text{buy}} = 0$ and $v = v^2$ with $v^1 < v^2$. Setting $\Delta H_0^{\text{buy}} = -1$ and $\tau = \tau^*$ for the first potential resident and $\Delta H_0^{\text{buy}} = 1$ and $\tau = -\tau^*$ for the second potential resident strictly increases the welfare objective if $v^1 + p_1^h - p_0^h \leq \tau^* \leq v^2 + p_1^h - p_0^h$.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.